Gab q Hia

AN EASY

INTRODUCTION

TO

PRACTICAL GUNNERY,

OR,

The Art of ENGINEERING.

CONTAINING,

- I. Decimal Arithmetic, Extraction of Roots.
- II. Menſuration of Planes and Solids, and the Computation of the Strength of Timber Joiſts, *&c.*
- III. The Computation of Balls and Shells.
- IV. The neceſſary Theorems in Geometry, demonſtrated in a very plain and eaſy Way.
- V. The Nature and Uſe of Logarithms.
- VI. Plane Trigonometry in all its Caſes; with the Method of taking Heights and Diſtances, and at one Station.
- VII. GUNNERY, where the Caſes are ſolved by Addition and Subtraction only, with numeral Examples, the Rules in Words at length, for thoſe who are unacquainted with the Elements of the higher Geometry.
- VIII. The Theory of Projectiles.
- IX. Tables of Experiments of Cannons and Mortars, with ſome Obſervations.
- X. The Solution of a difficult Problem to find the Velocity of a Bullet, ſhot from any Piece of Artillery.
- XI. The neceſſary Tables of Gunnery.

Illuſtrated with COPPER PLATES.

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THE

PREFACE.

A Great Part of the enſuing Treatiſe was ſome Years ago drawn up for my own Uſe in teaching ſome young Gentlemen, who had not a Conveniency or Opportunity to ſearch into different Authors that have treated on this Subject: The Particulars of the following Sheets are as follows;

I. *Decimal Arithmetic*, *Extraction of the Square and Cube Roots*, being neceſſary to all thoſe who are inclined to proceed regularly in attaining this uſeful Branch of Knowledge.

II. *Menſuration of Planes and Solids*, and the Method of computing the Strength of Timber Joiſts, whether Oak, Elm, Beech, or Fir; and it muſt be here ſuppoſed that the Timber is homogeneous, and of the ſame Goodneſs as thoſe Pieces Mr *Emerſon* made his Experiments with, or elſe a proper Allowance muſt be made for the Defect.

III. The uſual Rules for finding the *Weight and Diameters of Bullets, whether Lead or Iron*; the requiſite Charges of Powder for any Piece of Artillery of any given Dimenſions, and for eſtimating the Quantity of Powder for Bomb Shells.

IV. The neceſſary *Problems* and *Theorems* in the *Elements of Geometry*, demonſtrated in a very eaſy and ſhort Way, as may be comprehended by any Perſon, without the Help of a Maſter.

V. *Logarithmical Arithmetic*, ſhewing the Nature and Uſe of Logarithms; with *Plane Trigonometry* ſolved in all its Caſes, illuſtrated with proper Examples, the Uſefulneſs whereof is ſo great in all Parts of the

Mathematics, that the moſt important and uſeful Parts of Knowledge would be quite loſt, if Men were deſtitute of it; for the Engineer would make but a deſpicable Figure when he came to take the Meaſures of Heights and Diſtances, whether acceſſible or inacceſſible, or for laying down Plans of Fortification on the Ground, computing the Capitals, Curtains, Flanks, Faces, Lines of Defence, *&c.* if he was ignorant of *Trigonometry.*

VI. The Reader will find ſeveral Examples for *meaſuring Heights and Diſtances* and at one Station, all done by *Plane Trigonometry.*

VII. *The Theory of Practical Gunnery*, wherein the Doctrine of Projectiles is illuſtrated in a Multitude of the moſt uſeful Problems that can occur, with numeral Examples to each; and to make it more plain and uſeful to Perſons of but an ordinary Capacity, I have given the Rules to every Problem in Words at length, whether the Projections be horizontal, or thoſe made on aſcending or deſcending Planes; all which is deſigned more for thoſe Perſons who have not Skill and Abilities in the Mathematics, than for thoſe who are furniſhed with Mathematical Knowledge ſufficient to underſtand the learned Tracts on this Subject, and made farther Advances in the Elements of the higher Geometry. Yet,

VIII. For the Sake of my more ingenious Readers I have added what the celebrated Mr *Cotes* has wrote on Projectiles, done into *Engliſh* from his *Harmonia Menſurarum*, where they will find this Subject elegantly handled by this Author; to which I have annexed two general *Scholia* on Projectiles, of the learned Mathematician Mr *Emerſon*, taken from his *Mechanics* with his own Conſent, and illuſtrated them with Examples. I took this Method, becauſe many of the Problems in Gunnery were wrote before his *Principles of Mechanics* were publiſhed, and by theſe two *Scholia* the Solution of every Problem was tried, and the Anſwers very exactly agreed; ſo that I am in no Concern what ſome Perſons may ſay of this Performance; but how falſly and unjuſtly were his *Principles of Mechanics* cenſured and reprehended by a Set of Writers that did not underſtand the Subject, (is manifeſt from their own Performances) which is allowed by proper Judges to be the beſt Book of Mechanics that was ever publiſhed.

IX. The next Part treats of Experiments ranged into Tables; theſe Experiments were made with Cannon and Mortars, and Calculations were made thereon from the Principles of the immortal Sir *Iſaac Newton*, by *D. Bernoulli* and *M. Gunther*: This Memoir, quoted by Mr *Robins* in his *New Principles of Gunnery*, is printed in *Latin*, and was ſo communicated to me by my much honoured and eſteemed Friend Mr *Dodſon*, F. R. S. and Mathematical Maſter of *Chriſt*'s Hoſpital; a particular Account of this Memoir is inſerted at the Concluſion of this Work.

X. Having a little ſpare Room, I have given a Fluxionary Solution to a difficult Problem, for finding the Velocity of a Bullet ſhot from any Piece of Artillery; though this is of no material Uſe in Practical Gunnery, yet it perhaps may pleaſe ſome of my Readers, as the Concluſion comes out ſo ſimple.

XI. I have alſo inſerted the neceſſary Tables of Gunnery, *&c*

What is ſaid of the Flight of Bombs in the following Sheets, is equally applicable to Cannon Balls; the Method for directing Cannons, and placing them at any Elevation, is by means of proper Inſtruments for that purpoſe. And here I would not be underſtood to mean, that Experience does always exactly agree with the Rules given for throwing Bombs to hit an Object, for there often happens many unavoidable Accidents which will cauſe ſome Difference; and yet theſe Rules are not for that Reaſon to be careleſly attended to, for they furniſh the Gunner with the readieſt and ſhorteſt Methods of perfecting his Practice, and will anſwer the End propoſed. Indeed a careful Attention muſt be given to thoſe Perſons who are daily employed in the Exercife of Artillery, and have founded their Methods upon long Experience; for this will always be of more weight than the moſt learned Precepts. But, yet ſuch a Perſon will be better able to judge by the Help of theſe Rules, joined to his own Experience, than he can by his Experience alone.

As I have conformed to the Method of other Writers, it being the moſt eaſy and the beſt adapted for Beginners; ſo I expect to have Objections raiſed againſt me, becauſe I have made no Allowance for the Air's Reſiſtance; now to anticipate theſe Objections, What Engineer computes this for *Practical Engineering?* The Problem would then become very complicate, and of a very operoſe kind. The Taſk is not only extreme difficult, but uſeleſs; for to ſhew how far a Projectile deviates from a parabolic Track by the Reſiſtance of the Air, is one of the moſt intricate Problems in all the Doctrine of Fluxions; and which is worſe ſtill, it can be of no manner of uſe; for the Motion of a Projectile in a reſiſting Medium can be reduced to no Rules, ſo as to be of any uſe in the Art of Gunnery. I thought therefore, by laying down ſhort and plain Rules for Practice, it would be more ſerviceable to the young Engineer, than to puzzle him with tedious Demonſtrations and perplexed Calculations in Fluxions, which would create an endleſs deal of Trouble for no real Advantage.

It perhaps may be ſaid, I might have added ſome Problems by ſuppoſing the Shot to move in a right Line to a point blank Diſtance from the Mouth of the Piece, and then to deſcribe the Curve of the Parabola; but I do not apprehend it can be of any uſe in Gunnery by putting a right Line and Parabola together; for the Curve deſcribed by a Ball on this

Hypotheſis, will be nothing like the Curve deſcribed in the Air, and therefore I have purpoſely omitted it.

But if any one would make a Computation from the Reſiſtance of the Air, he will find the Problem elegantly ſolved by Fluxions at Page 84, Vol. II. of my *Miſcellanea Curioſa Mathematica*, where it is ſaid, that the Deſcription of the true Curve is ſo perplex, that it is ſcarcely of any uſe; and if according to the ſecond Corollary the Reſiſtance be ſmall, as is commonly experienced by an Iron Ball projected thro' the Air with a ſufficient Velocity, the Curve which the Globe deſcribes in a reſiſting Medium, does not differ much from a Conic Parabola; yet becauſe the Reſiſtance diminiſhes the Velocity of the Projection, the Ordinate to the Trajectory in a reſiſting Medium will be a little leſs than the Ordinate to the Conic Parabola; but notwithſtanding this, the Problem, by reaſon of its difficulty, is of no uſe in Gunnery.

The *Art of Gunnery* is a Science at all times uſeful, and much more ſo amongſt the *Engliſh*, who I am ſure have as great an Inclination to Arms and Arts as any Nation in *Europe*; their matchleſs Valour and Bravery, their ſingular Prudence and Conduct, both by Sea and Land, is ſo univerſally known, that many powerful States have been conſtrained to yield to them by the Dint of their Sword; and as to Arts and ingenious Literature, they juſtly claim a true Title to the Empire of human Knowledge.

Theſe Sciences, it is known to all, are the peculiar Care of Princes, that deſign to raiſe the Force and Power of their Countries; and theſe are none of the leaſt Arts whereby the *French* King hath brought his Subjects to make that Figure which they at this time do; I mean, the Care that he takes for educating thoſe appointed for this Service; he orders that there be Profeſſors to teach theſe Sciences publicly

licly in ſeveral Parts of the Kingdom, that the Teachers muſt know Deſigning, and to teach it to their Pupils, in order to lay down the Appearances of Things in their real Form and Situation ; they are to keep their Schools open, and to read four times a Week to their Scholars, where they muſt have Books and Inſtruments neceſſary to teach their Art, who have handſome Salaries from the Government for that Service, and to teach *gratis*. The Directors of Hoſpitals are obliged to ſend to theſe Academies every Year ſeveral of their Boys, to be taught and furniſhed with Books and Inſtruments, explained with a vaſt Variety of Experiments, and thereby Practice and Theory go on hand in hand, and receive mutual Aſſiſtance from each other; and that nothing can exceed the Order of theſe Schools, the Officers placed at the Head of them are of the greateſt Ability and Knowledge in the Management of Artillery, which they teach with as much Method as Grammar and Accompts are taught in our Schools ; and hence it is that *France* is well provided with ſo great a Number of able and ſufficient Engineers.

During the Time of the late unnatural Rebellion, ſome Mathematical Lectures were read in *London* to inſtruct Officers in the Art of Engineering and Fortification, as knowing very well the Importance thereof, and this continued ſome time after the Peace ; and it is worthy the Conſideration of the Legiſlature, whether the reſtoring and continuing this even in times of Peace, be not expedient for the bringing up Engineers, who are ſo uſeful and valuable in the Military Way, and ſo difficult to be had in time of War, and ſo little dangerous in time of Peace.

The Arts of War and Trade require much the Aſſiſtance of theſe Sciences, an Officer that underſtands Fortification and Engineering, will, *cæteris paribus*, much better defend his Poſt, and knowing wherein its Strength conſiſts, or make uſe of his Advantage to his Enemies ruin, than he that does not; he knows when he leads never ſo ſmall a Party, what his Advantage and Diſadvantage in defending and attacking are, and hereby can do more Service than another of as much Courage, who for want of ſuch Knowledge, it may be, throws away himſelf, and a Number of brave Fellows under his Command, and it is well if the Miſchief reaches no further.

Fortification is ſo neceſſary an Art, that it greatly deſerves more Encouragement; for, excepting two or three Academies founded on Royal Authority, I cannot learn that it is properly taught in any Parts of this Kingdom; for moſt of thoſe Gentlemen that make Geometry their peculiar Province in teaching it, apply it often to uſeleſs Speculations. Now Fortification is the chief Art wherein Geometry is uſed, and to it only is owing the great Succeſs and Improvement of Military Architecture; indeed, where the Ground is regular, it admits but of ſmall Variety; yet where the Ground is made up of natural Strengths and Weakneſſes, it affords ſome Scope for Thinking and Contrivance; and what contributes much towards its Strength, with regard to its Force or Reſiſtance, are the due Meaſures and Proportions of its Lines and Angles; as it is now arrived to ſuch a Degree of Perfection, that in all likelihood it cannot receive any eſſential Addition; ſo it is the more eaſily to be comprehended; becauſe, it ſeems impoſſible to find any thing that is not already invented for the making Attacks upon Places. Earth is made uſe of for Trenches, Banks and Retrenchments; Water for Sluices and Inundations; and Fire is applied various Ways; below, by the Contrivance of Mines and their Galleries, whoſe Havoc is dreadful, and Deſtruction rapid, in a Moment burſting open the Bowels of the Earth, rending in Pieces the ſtrongeſt Walls, the firmeſt Towers, and blowing up whole Troops of

Men; above, by Bombs, Carcaſſes, and Granadoes, directly by Cannons, Mortars, &c. "Let the "Reader imagine to himſelf (ſays an ingenious "Writer) a great Number of Mortars playing con- "tinually from the Batteries round a beſieged Town, "and Bombs like one uninterrupted Tempeſt of "Fire falling at once on all its Quarters, forcing "their Paſſage irreſiſtibly, demoliſhing Buildings, "and burying Multitudes under the Ruins, tearing "up Pavements, ſweeping whole Ranks of Men "away, and toſſing their mangled Limbs into the "Air, penetrating with the Violence of Lightning "even ſubterraneous Vaults, overthrowing, rending "or ſetting on Fire whatever they touch, their Ex- "ecution is experienced as fatal and general, as "their Effects are terrible and deſtructive." The Cunning of Mankind never exerting itſelf ſo much as in their Arts of deſtroying one another; therefore Methods of Defence have been invented to prevent in a great Meaſure theſe Effects, and this is the proper Buſineſs of Fortification; and unleſs Mankind find out ſome other Elements, there is not much Probability that they can invent new Methods, either offenſive or defenſive.

It were to be wiſhed that proper Encouragement were allowed for experienced Mathematicians and Engineers to make all the neceſſary Experiments on Pieces of Artillery, for this Art now makes one of the moſt conſiderable Branches of the military Science, and it is abſolutely neceſſary to have it underſtood by many, in order to ſupport the Honour of our Arms, and to maintain the public Safety of this Nation; and I am ſure never more ſo than at this preſent Conjuncture, where it is ſaid from public Authority that *Great Britain* or *Ireland* is daily threatned to be invaded by one of the moſt powerful and formidable Fleets and Armies that *France* ever had; whoſe Armies are in continual Motion on all Sides, their

Coaſts towards the Ocean ſwarm with Troops, their Roads to *Flanders*, *Normandy* and *Brittany*, are continually covered with Carriages laden with Cannon, Warlike Stores, Arms of all kinds, Proviſions, *&c.* all the *Apparatus* of ſome great Enterprize, whoſe Injuſtice as well as Inſolence is circumſcribed by no Bounds; who now fully ſhew their ambitious Projects, ill Faith, notorious Breach of Treaty, and manifeſt Aggravation againſt the beſt of KINGS that ever ſat on the *Britiſh* Throne. This Attempt of invading theſe Kingdoms in revenge for his Majeſty's generous and ſteady Conduct in maintaining the Rights and Poſſeſſions of his Crown and Subjects, againſt the unprovoked Aggreſſions and Hoſtilities firſt begun on the Part of *France*, is ſo unjuſt, daring and inſolent, as not to be paralleled in any Hiſtory. But—

To conclude, as the Practice of *Sea Gunnery* is extreme eaſy, their Marks being moſtly point blank or very near it, and that Aſcents and Deſcents bear no conſiderable ſhare here, I have taken no notice of it, and have omitted the Demonſtrations of the Rules in Gunnery in order to render the Whole as conciſe, and deliver it with as much plainneſs as my own Abilities and the Nature of the Subject would permit me; and I hope the candid and impartial Reader will excuſe what is amiſs, and amend thoſe Errors which unavoidably attend things of this Nature, or may happen by reaſon of the Author's great Diſtance from the Preſs; for he has no other Deſign in publiſhing this Book, than to be ſerviceable to his own Countrymen, for whoſe Benefit he heartily wiſhes it was much more compleat than it is.

Sept. 4, 1756.

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ERRATA.

Page 57, Step 2, read $\angle C+\angle DBC=\angle ADB$.

—— 62, Step 3, dele 1.

—— 67, third Line from the Bottom, after VA, read *by Theo.* 11.

—— 70, Step 6, for $CD+AC+$ *&c.* read $CD\times AB+$*&c.*

—— 106, laſt Line but one, read $\angle ACD=23^{\circ}\ 20'$.

—— 110, Line 14, read $\angle DBC=58^{\circ}\ 2'$.

—— 112, laſt Line but one, read 9.74473.

—— 120, for *The Momentum which conduces,* read *The Momentum or that which conduces*

—— 164, Line 25, for $x=AE$, read $x=AB$.

OF

DECIMAL ARITHMETIC.

DEFINITIONS.

A Fraction is called *Vulgar* or *Decimal*, according as it is divided, as $\frac{3}{5}$ is called a *Vulgar Fraction*, the uppermoſt Number is called the *Numerator*, and the undermoſt the *Denominator*.

A *Decimal* Fraction ſuppoſes *one* of any thing whatever to be divided into 10 Parts, 100 Parts, 1000 Parts, *&c.* and hath always a Dot ſet before it as .5 is called 5 Tenths, and in Vulgar Fractions $\frac{5}{10}$, but as Decimals are the eaſieſt I ſhall ſay nothing of Vulgar Fractions here.

As Places in Whole Numbers increaſe by Tens, ſo do Places in Decimals decreaſe by Tens; for as the firſt, ſecond, third, *&c. Place* above that of Units is Tens, Hundreds, Thouſands, *&c.* ſo the firſt, ſecond, third, *&c. Place* below that of Units, is *Tenths*, *Hundredths*, *Thouſandths*, decreaſing in a tenfold Proportion, as is manifeſt from the following Table.

7 Millions.
6 Hundreds of Thouſands.
5 Tens of Thouſands.
4 Thouſands.
3 Hundreds.
2 Tens.
1 Units.
.2 Tenth Parts.
3 Hundredth Parts.
4 Thouſandths.
5 Ten Thouſandths.
6 Hundred Thouſandths.
7 Millionths.

So { .5 / .25 / .175 } ſignifies { 5 Tenths. / 25 Hundredths. / 175 Thouſandths. }

Cyphers ſet after Decimals do not alter their Value, as .8 and .8000 are the ſame, *viz.* only 8 Tenths: But when Cyphers are ſet before Decimals, they remove them farther from the decimal Point, and therefore decreaſe them, as .08 is leſs than .8, ſo .0008 is leſs than .008, conſequently, each removal of a Figure into a Place forward makes it ten times leſs than it was before.

To reduce Vulgar Fractions into Decimals.

RULE.

Add as many Cyphers to the Numerator as you intend Decimals, and divide by the Denominator, the Quotient is the Decimal required.

Examples.

1. Reduce $\frac{1}{4}$ of any thing into a Decimal.

4) 1.00 (.25 the Decimal.
8
——
20
20
—

2. Reduce $\frac{1}{2}$ of any thing into a Decimal.

2) 1.0 (.5 the Decimal.
1 0
——

3. Reduce $\frac{3}{4}$ of any thing into a Decimal.

4) 3.00 (.75 the Decimal.
2 8
——
20
20
—

So the Quarter, Half, and three Quarters of any thing are .25; .5; and .75 Decimally.

4. Reduce $\frac{2}{7}$ into a Decimal.

```
7 ) 2.00000 ( .28571 the Decimal of 2/7.
    14
    ---
     60
     56
     ---
      40
      35
      ---
       50
       49
       ---
        10
         7
        ---
         3
        ---
```

5. Reduce $\frac{1}{32}$ into a Decimal.

```
32 ) 1.00000 ( .03125
      96
     ---
      40
      32
     ---
       80
       64
      ---
       160
       160
      ---
```

Note, Here I annexed five Cyphers to 1 the given Numerator, as I intended to have five Places in Decimals, and there ariſe but four in the Quotient, I muſt ſupply ſuch Defect by prefixing as many Cyphers on the left Hand of the firſt Figure in the Quotient as there want Places; as in the Example above there is prefixed a Cypher to make the Quotient compleat, becauſe I annexed five Cyphers to the Numerator.

6. Reduce 6*s*. 8*d*. into the Decimal of a Pound.
Thus 6*s*. 8*d*. is = 80*d*. and a Pound = 240 *d*.
Then 240) 80.00000 (.33333 is the Decimal.

7. What is the Decimal of 5*C.* 1*Qr.* 11*lb.* the Integer being a Tun Weight?

Thus a Tun Weight=2240*lb.* and 5*C.* 1*Qr.* 11*lb.*=599*lb.*

Then 2240)599.0000(.2674 the Decimal.

To find the Value of any Decimal Fraction.

RULE.

Multiply your given Decimal by the Parts of the next inferior Denominator that is equal to the Integer, the Decimal dotted off gives the Part thereof; and if there be any Remainder, multiply it by the next inferior Denomination, &c. till all is done.

Examples.

1. What is the Value of .875 of a Pound Sterling?

Thus £ .875
20
s. 17.500
12
d. 6.000

Anſwer 17*s.* 6*d.*

2. What is the Value of .9874 £?

Thus .9874 £
20
19.7480
12
8.9760
4
3.9040

Anſwer 19*s.* $8\frac{3}{4}$.904 *d.*

3. What is the Value of .5875 of a Tun weight?

Thus .5875 *Tun.*
20
11.7500
4
3.0000

Anſwer 11C. 3*Qrs.*

4. What is the Value of .3712 of a Hundred weight?

```
Thus          .3712 C.
                  4
              -------
             1.4848
                 28
              -------
              38784
              9696
              -------
            13.5744
                 16
              -------
              34464
              5744
              -------
              9.1904        Anſwer.
              -------   1 Qr. 13 lb. 9.1904 oz.
```

ADDITION of DECIMALS.

RULE.

LET the Numbers be placed according to their Value, as Units under Units, Tens under Tens, *&c.* in Whole Numbers; and in Decimals Tenths under Tenths, Hundredths under Hundredths, *&c.* then work as in Whole Numbers, always remembering to make as many Places of Decimals in the Sum total, as the greateſt Number thereof were in your Example.

Examples.

£	*Yards.*	818.4 £
.584	48.94	6.97
.371	21.648	3.81
.281	6.5	.54
.846	19.8168	.61
.75	72.1108	1.82
.6	16.614	3.25
£3.432	185.6296	835.40

Or £3 : 8 : 7½.72 Or 185 *Yds.* 2 *Qrs.* 2 *Na.* &c. Or £835 : 8

SUBTRACTION.

Place your Numbers as directed in Addition, then work as in Whole Numbers.

Examples.

From	.591	.6198	1.	54.
Take	.397	.1499	0.987	6.9843
Remains	.194	.4699	0.013	47.0157

MULTIPLICATION.

Work in Multiplication as in Whole Numbers, always remembering to ſet off as many Decimals in the Product as there are Decimals both in your Multiplicand and Multiplier, and if you have not a ſufficient Number in your Product, then prefix as many Cyphers as you want.

Examples.

.3416	.4918
.37	37
23912	34426
10248	14754
.126392	18.1966
37.241	.131461
12.23	.2132
111723	262922
74482	394383
74482	131461
37241	262922
455.45743	.02802748 52

DIVISION.

Divide as if all were Whole Numbers, annexing Cyphers to the Dividend if need be; and when you have finiſhed the Operation, point off for Decimals ſo many Places as the decimal Parts uſed in the Dividend exceed thoſe of the Diviſor, and thoſe to the Left, if any, are Whole Numbers.

If the Places in the Quotient are not as many as the above Rule requires, then ſupply the Defect with Cyphers on the left Hand of the Quotient.

Examples in all the Varieties.

```
246 ) 160.884 ( .654 Quotient.
      147 6
      -----
       13 28
       12 30
       -----
          984
          984
          ---

246 ) 16.0884 ( .0654
      14 76
      -----
       1 328
       1 230
       -----
         984
         984
         ---

2.46 ) 1608.84 ( 654
       1476
       ----
        1328
        1230
        ----
          984
          984
          ---
```

```
2.46 ) 16.0884 ( 6.54
       14 76
       ------
        1 328
        1 230
        ------
          984
          984
         -----

2.46 ) 160884.00 ( 65400
       1476
       -----
       1328
       1230
       -----
         984
         984
        -----
           00
          ---

.0246 ) 160.8840 ( 6540
        147 6
        -----
         13 28
         12 30
         -----
           984
           984
          -----
             0
             -

.0246 ) .160884 ( 6.54
         1476
         -----
          1328
          1230
          -----
            984
            984
           -----
```

The

The Rule of Three in Decimals.

State your Queſtion as in Whole Numbers, and work as aforeſaid.

Examples.

If a Pound and a Half of Butter coſt a Groat, what will two Pounds coſt?

Thus

lb. — *d.* — *lb.*

If 1.5 ——— 4 ——— 2

$$\begin{array}{rrl}
 & 2 & \\
 & 4 & \\
\hline
 & & d. \\
1.5) & 8.000 (& 5.33 \\
 & 75 & 4 \\
\hline
 & 50 & 1.32 \\
 & 45 & \\
\hline
 & 50 & \\
 & 45 & \\
\hline
 & 5 & \\
\hline
\end{array}$$

Anſwer $5\frac{1}{4}$.32*d.*

Two Men bartered, *A* had 40.7 Yards of Shalloon for which *B* gave him 25.6 Ells of Holland, at 4*s.* 6*d.* ⅌ Ell, what is the Shalloon a Yard?

Ell. — *s.* — *Ells.*

If 1 ——— 4.5 ——— 25.6

$$\begin{array}{r}
25.6 \\
4.5 \\
\hline
1280 \\
1024 \\
\hline
s.\ 115.20 \\
\hline
\end{array}$$

Yards. *s.* *Yard.*
If 40.7 ——— 115.2 ——— 1

1

———

s.

40.7) 115.200 (2.83

814 12

——— ———

3380 *d.*9.96

3256 4

——— ———

1240 3.84

1221

———

Anſwer 2*s.* 9¾8*d.* ℘ *Yard.* 19

—

A owes *B* £ 296 : 17, but not being able to pay the Whole he compounds for 7*s.* 6*d.* in the Pound, what muſt *B* receive for his Debt?

Firſt the Decimal of 17*s.* is .85 £ then

£ *s.* £
If 1 ——— 7.5 ——— 296.85

7.5

———

148425

207795

———

s. 2226.375

12

———

4.500

4

———

2.000

———

Anſwer 2226*s.* 4½.6*d.*
or £ 111 : 6 : 4½.

Note, When more requires leſs. that is, when the third Term is greater than the firſt, and requires the fourth Term to be leſs than the ſecond, or when leſs requires more, that is, when the third Term is leſs than the firſt, and requires the fourth Term to be greater than the ſecond, then the Proportion is *inverſe*; and to find the fourth Term

Multiply the firſt and ſecond Terms together, and divide that Product by the third Term, the Quotient is the fourth Term or Anſwer to the Queſtion.

Examples.

There is Proviſion enough in a Caſtle to ſerve a Garriſon of 200 Soldiers for a Year, and it was reported that an Enemy were going to approach it, the Garriſon was augmented to 500 Men more, how many Days will the Proviſion ſerve theſe 500 Men?

Men. *Days.* *Men.*
If 200 —— 365 —— 500
200
——
5|00) 730|00(
——
146 *Days Anſwer.*

A certain Garriſon, containing 4000 Men, hath Proviſion ſufficient for ſix Weeks and four Days, but being about to be beſieged by the Enemy, and no Aſſiſtance to be got to relieve it in leſs than two Months; how many Men muſt the Governor diſcharge out of the ſaid Garriſon, that the ſaid Proviſion may ſerve the Remainder four Months two Weeks?

Firſt the Decimal of 4 Days, a Week the Integer, is .57, For 7)4.00(.57 and the Decimal of 2 Weeks, a Month the Integer, is .5. Then,

Weeks. *Men.* *Weeks.*
If 6.57 —— 4000 —— 16.5
4000
——
16.5)26280.00(1592.7 the Number will ſerve.
165
——
978
825
——
1530
1485
——
450
330
——
1200
1155
——

Then from 4000
Take 1592
——
Remains 2408 Men to be diſcharged.

Compound Rule of Three Direct.

In the Compound Rule of Three direct there are five Numbers given to find a ſixth, which is to the third as the Product of the two laſt to the Product of the two firſt.

Multiply the firſt and ſecond together for a Diviſor, and the fourth, fifth, and third Terms together for a Dividend, the Quotient ariſing by dividing the Dividend by the Diviſor is the Anſwer.

Example.

If 12 Men in 8 Days dig 48 Cubic Fathoms of Earth to make an Entrenchment; how many Fathoms at the ſame Rate will 28 Men dig in 4 Days?

	Men.	*Days.*	*Fath.*	*Men.*	*Days.*
If	12 ——	8 ——	48 ——	28 ——	4
	8			4	
	96 Diviſor.			112	
				48	
				896	
				448	
			96)	5376(	56 Fathoms. *Anſ.*
				480	
				576	
				576	

Compound Inverſe Rule of Three.

Is to find a ſixth Number from five given Numbers, which is to the third as the Product of the two firſt is to the Product of the two laſt. To find the ſixth, Multiply the Product of the two firſt by the third, and this Product divide by the Product of the fourth and fifth, the Quotient is the ſixth, or Anſwer.

Example.

Suppoſe there be 4000 Men in a Garriſon beſieged, and their daily Allowance is 20 Ounces of Bread out of the Magazine for 3 Months together, but the Governor finding the Siege is likely to continue longer, and perceiving he had loſt by this Time 1000 brave Men, he is determined to hold out 6 Months, by huſbanding his Stock of Proviſions to the beſt Advantage; how much Bread muſt each Man be allowed a Day?

	Men.	*Months.*	*Ounces.*	*Men.*	*Months.*
If	4000 ——	3 ——	20 ——	3000 ——	6
	3			6	
	12000			18000	
	20				

18|00)240|000(13$\frac{1}{3}$ Ounces for each Man a Day.
18
60
54
6

Extraction of the Square Root.

ANY Number multiplied into itſelf is ſaid to be ſquared, or the Square of that Number: thus 6 multiplied into itſelf, or 6 × 6 = 36 the Square of 6, whence the following Table.

Roots	1.	2.	3.	4.	5.	6.	7.	8.	9
Squares	1.	4.	9.	16.	25.	36.	49.	64.	81.

RULE.

When any Number is given to be extracted, point it off in every other Place, beginning at Unity; then find the firſt Figure in the Root, whoſe Square ſhall be equal to, or neareſt leſs than the Figure or Figures to the firſt Point, and then ſubtract that Square from the firſt Point, and to the Remainder bring down the next Point, and call that the Reſolvend. Then double the Quotient or Root, and ſet it for a Diviſor on the left Hand of the Reſolvend, and ſeek how oft the Diviſor is contained in the Reſolvend, reſerving always the Units Place, and put the Anſwer in the Quotient or Root, and alſo in the Units Place of the Diviſor, then multiply the Diviſor by the Figure laſt put in the Quotient or Root, and ſubtract the Product from the Reſolvend as in common Diviſion, and bring down the next Point to the Remainder, and this is the Reſolvend, and proceed as before till the Operation is finiſhed.

Examples.

What is the Square Root of 116964?

Firſt let it be pointed thus 11̇69̇64̇, then find the neareſt Square Number to 11 the firſt Period, as 9, &c. Thus

```
     11̇69̇64̇ ( 342 Root.              For 342
     9                                by 342
     ——                               ——————
  64 ) 269 Reſolvend.                    648
       256                              1368
       ———                             1026
 682 ) 1364 Reſolvend.                 ——————
       1364                            116964 Proof.
       ————                            ——————
          0
```

What is the Square Root of 13169641?

```
Thus,    13̇1 69̇64̇1̇ ( 3629 Root.
          9
         ——
     66) 416
          396
          ———
    722 ) 2096
           1444
          —————
   7249 ) 65241
           65241
          ——————
               0
```

When any whole Number is propoſed to extract the Square Root, and there be a Remainder, the Root may be found to any Degree of Exactneſs, by adding two Cyphers to the Remainder, and proceeding as before.

What is the Square Root of 37468?

$\dot{3}7\dot{4}6\dot{8}$ (193.566, &c.
1

29) 274
261

383) 1368
1149

3865) 21900
19325

38706) 257500
232236

387126) 2526400
2322756

203644, &c.

When a Mixt Number or Decimal is to be extracted, point every other Figure as before, beginning at the Units Place in whole Numbers, but in the Decimal always begin in the ſecond or hundreds Place, and if you have not a full Period then add a Cypher to compleat it, and when you have not as many Figures in the Root as Dots or Periods, then prefix a Cypher or Cyphers to anſwer them.

What is the Square Root of 523.176?

$\dot{5}2\dot{3}.1\dot{7}6\dot{0}$) 22.87, &c.
4

42) 123
84

448) 3917
3584

4567) 33360
31969

1391, &c.

What is the Square Root of .143219?

$.1\dot{4}3\dot{2}1\dot{9}$ (.3784, &c.
9
67) 532
469
748) 6319
5984
7564) 33500
30256
3244, &c.

What is the Square Root of 7?

$\dot{7}.0\dot{0}0\dot{0}0\dot{0}$ (2.645, &c.
4
46) 300
276
524) 2400
2096
5285) 30400
26425
3975, &c.

What is the Square Root of .0007612816?

$0\dot{0}0\dot{7}6\dot{1}2\dot{8}1\dot{6}$ (.02759
4
47) 361
329
545) 3228
2725
5509) 50316
49581
735

To find the Square Root of a Vulgar Fraction, reduce it into a Decimal, and extract the Square Root of that Decimal, and the thing is done.

What is the Square Root of $\frac{4}{8}$ or $\frac{1}{2}$?

Thus $\frac{4}{8}$ or $\frac{1}{2}$ in Decimals is .5 then $.5\dot{0}0\dot{0}$ (.707, &c.

49

1407) 10000

9849

151, &c.

To extract the Cube Root.

RULE.

Firſt point off from the Unity Place every *third* Figure, find the firſt Figure in the Root by the following Table, ſubtract it from the firſt Period, augment the Remainder by the firſt Figure in the next Point, this call the Dividend; divide this Dividend by the triple Square of the whole Root laſt found, the Quotient Figure found annex to the Root; Cube the Root thus found, and ſubtract it from as many Points as you have brought down; to the Remainder bring down the firſt Figure of the next Point for a new Dividend, divide this new Dividend by the triple Square of the Root, the Quotient Figure annex to the Root again, Cube the whole Root, and ſubtract it from as many Points as you have brought down, and proceed in this manner till the Work is ended.

The TABLE of POWERS.

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81
Cubes	1	8	27	64	125	216	343	512	729

Thus to extract the Cube Root of 13312053, the Number is firſt to be pointed after this manner, *viz.* 13̇312̇053̇, then you are to write in the Root the Figure 2, whoſe Cube 8 is the next leſs Cube to the Period 13, which is not a perfect Cube Number, and having ſubtracted that Cube according to the Rule, there will remain 5, which being augmented by the next Figure of the Reſolvend 3, and divided by the triple Square of the Root 2 or 12, by ſeeking how oft 12 is contained in 53, it gives 4 for the ſecond Figure of the Root, but ſince the Cube of 24, *viz.* 13824 would come out too great to be ſubtracted from 13312 that preceeds the ſecond Point there muſt only 3 be wrote in the Root, then 23 Cubed is 12167 which taken from 13312 will leave 1145, which augmented by the next Figure of the Reſolvend 0, and divided by the triple Square of the Quotient 23, *viz.* 1587, it gives 7 for the third Figure of the Root; then 237 Cubed and ſubtracted from the three Periods or the whole Reſolvend, there remains 0. Whence the true Root is 237. *See Sir* Iſaac Newton's *Arithmetica Univerſalis.*

Examples.

13̇312̇053̇ (237 Root.
8

2×2=4×3=12) 53 (4 or 3.

Subtract Cube 12167

23×23=529×3=1587) 11450 (7

Subtract Cube 13312053

Remains 0

What is the Cube Root of 27054036008?

27̇054̇036̇008̇ (3002 Root.
27

3×3=9×3=27) 0542 (2

3002 Cubed is 27054036008

0

What is the Cube Root of 122615327232?

$12\dot{2}61\dot{5}32\dot{7}23\dot{2}$ (4968 Root.
64

4×4=16×3=48) 586 (9

49 Cubed, Subtract 117641

49×49=2401×3=7203) 49743 (6

Subtract 496 Cubed, *viz.* 122023936

496×496=246016×3=738048) 5913912 (8

Subtract the Cube of 4968, *viz.* 122615327232

Remains 0

When the Reſolvend has a Remainder, you may carry on the Root to any Degree of Exactneſs by adding 3 Cyphers for each new Period you intend to carry the Root to in Places of Decimals; and always remember to point off every third Figure from the Tenths Place in Decimals, the whole Numbers as before.

What is the Cube Root of 61218.00121?

$6121\dot{8}.00\dot{1}21\dot{0}$ (39.41 Root.
27

3×3=9×3=27) 342 (9

39 Cubed 59311

39×39=1521×3=4563) 19070 (4

394 Cubed 61162984

$394^{\mathit{Sqd.}}$=155236×3=465648) 550172 (1

3941 Cubed, is 61209566621

Remainder 8434589

What is the Cube Root of .0069761218 ?

.006976121800000 (.19107

1

1×1=1×3=3) 59 (9

19 Cubed 6859

19×19=361×3=1083) 1171 (1

191 Cubed 6967871

191×191=36481×3=109443) 82508 (0

1910 Cubed 6967871000

1910*Sqd.*=3648100×3=10944300) 82508000 (7

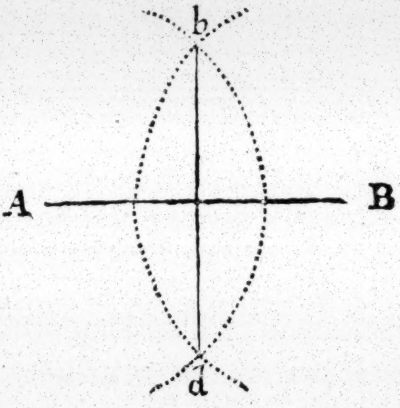
19107 Cubed, is 6975534818043

Remainder 586981957

PROBLEM I.

TO divide a given right Line into two equal Parts, *i. e.* To biſect a right Line.

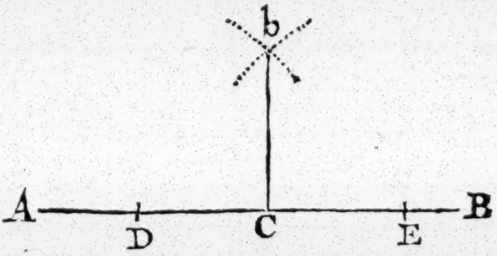
Let the given Line be A B, then with one Foot of your Compaſſes in A, opened to any Diſtance greater than half the Line A B, deſcribe an Arch, then with the ſame opening of your Compaſſes ſet one Foot in B, deſcribe another Arch, and where theſe two Arches cut one another, as in *b* and *d*, if the Line *b*, *d* be drawn that will biſect the given Line A B.



PROBLEM II.

From a Point in a given Line to raiſe a Perpendicular.

Let A B be the given Line, and the given Point be C, take two Points equally diſtant from C, as D E, and ſetting the Foot of your Compaſſes in D and E, opened to any Diſtance greater than AC, and ſtrike two Arches as above, and where the two Arches interſect one another as at *b*, there draw *b* C, that ſhall be the Perpendicular required.

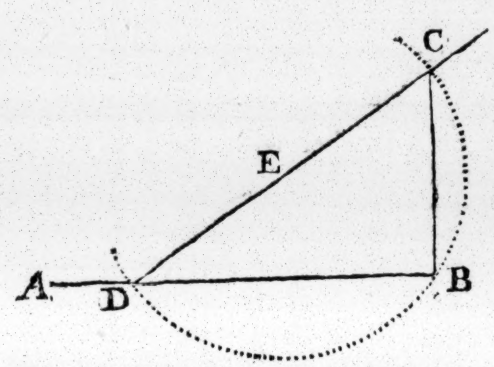


PRO-

PROBLEM III.

From the End of a given Line to erect a Perpendicular, as ſuppoſe at B.

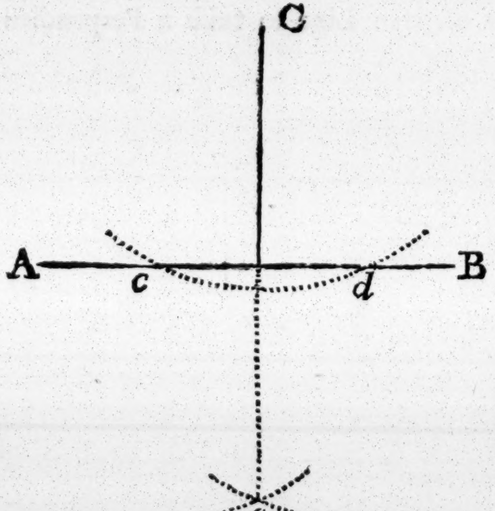
On any convenient Diſtance out of the given Line as at E, ſet one Foot of the Compaſſes and extend the other to the given Point B, with this Diſtance draw a Circle, and from the Interſection in the given Line as at D, draw a Line through the Point E to interſect the Arch above as C, then C B drawn will be the Perpendicular.



N. B. By this Problem may as many Perpendiculars as you pleaſe be raiſed from given Points on a right Line, as well as from the End.

PROBLEM IV.

To let fall a Perpendicular on any given Line, from a given Point above that Line, as ſuppoſe from C.



From the Point C deſcribe an Arch as *c d* on the Line A B, then with the ſame Diſtance ſet one Foot in *c* and *d* deſcribe two Arches below, and where they interſect as at *e*, draw C *e*, it will be the Perpendicular required.

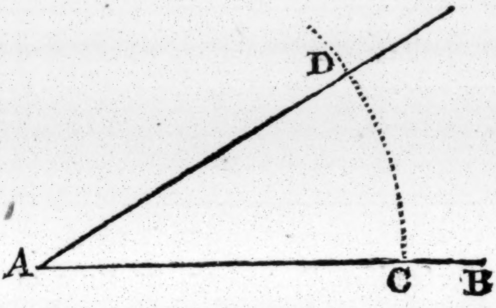
PROBLEM V.

With a given right Line to make an Angle of any Number of Degrees.

Example I.

Let it be required to lay down an Angle of 30° 30′ on A B from the Point A.

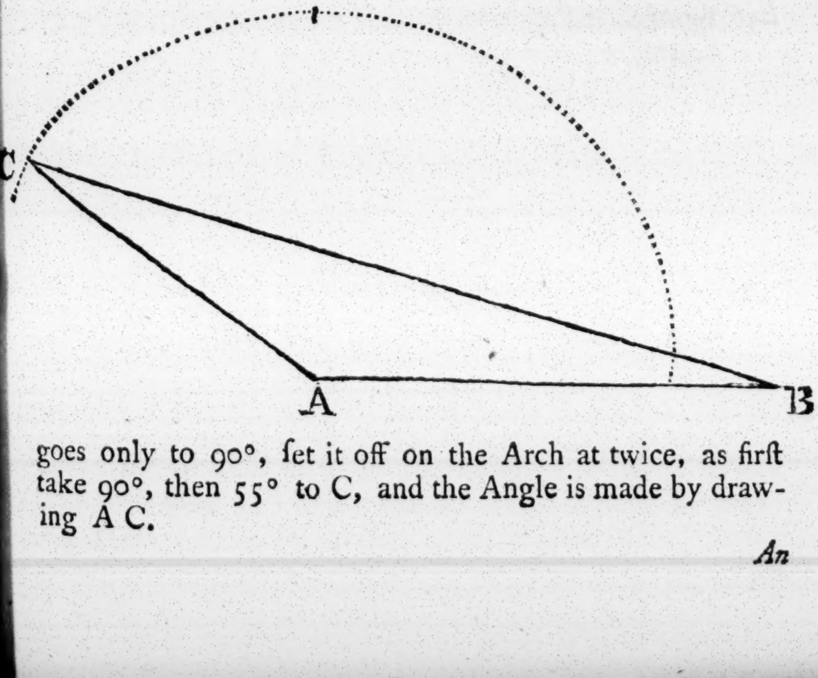
Take the Diſtance of 60 Degrees with your Compaſſes from the Line of Chords, then ſetting one Foot in A, with that Diſtance deſcribe an Arch as C D, then take 30° 30′ from the Line of Chords and put it from C to D, then a Line drawn from A through D the Angle D A C is the Angle required.



Example II.

To lay down an Angle above 90 Degrees.

From A on a given Line A B it is required to make an Angle of 145°. With the Diſtance of 60° on the Line of Chords ſweep an Arch, then ſince the Line of Chords goes only to 90°, ſet it off on the Arch at twice, as firſt take 90°, then 55° to C, and the Angle is made by drawing A C.



An Explanation of the SYMBOLS *uſed in this Treatiſe.*

$+$ ſignifies Addition, as $A+B$ denotes B added to A.

$-$ ſignifies Subtraction, as $A-B$ denotes B to be taken from A.

$\times$ ſignifies Multiplication, as $A\times B$ denotes A multiplied by B.

$=$ ſignifies Equality, as $A=A$ or A is equal to A.

$\surd$ ſignifies the Square Root, as $\sqrt{A}$ the Square Root of A, or $\sqrt{144}$ the Square Root of 144 or 12.

Log. ſignifies the Logarithm of.

$:$, $::$ ſignifies Proportion, or Rule of Three, thus $2:4::6:12$, that is, as 2 is to 4 ſo is 6 to 12.

$\triangle$ ſignifies Triangle, as $\triangle ABC$ denotes the Triangle ABC.

$\angle$ ſignifies Angle, as $\angle A$ ſignifies the Angle A.

s. ſignifies Sine, as *s.* ABC denotes the Sine of the Angle B.

t. ſignifies Tangent, as *t.* ABC denotes the Tangent of Angle B.

$\overline{\quad}|^{2}$ ſignifies the Square, as $\overline{CD}|^{2}$ denotes CD ſquared.

▭ ſignifies Parallelogram, as ▭ DB the Parallelogram DB.

$^{\circ}$ ſignifies Degrees, as 24° ſignifies 24 Degrees.

$'$ ſignifies Minutes, as $35'$ denotes 35 Minutes.

I. *To meaſure a Square.*

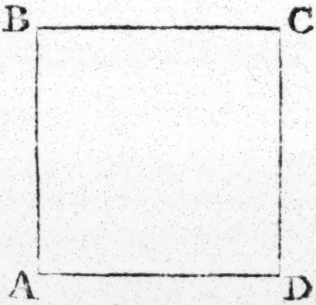
RULE.

SQUARE the Side, the Product is the Area, in ſuch a Name as you multiplied your Side by.

N. B. 144 Square Inches make 1 Square Foot.
9 Square Feet a Yard Square.
4840 Yards an Acre.

Examples.

Let the annexed Figure denote a Square as ABCD, whoſe Side A D is 79 Feet long, how many Square Yards the Content?



79
79
———
711
553
———
9)6241
———
Yds. 693 : 4 *Feet. Anſwer.*

If the Side of a ſquare Cloſe be 81¾ Yards on a Side, how many Acres doth the Cloſe contain?

Firſt, The Decimal of ¾ = .75

```
                      81.75
                      81.75
                     ------
                      40875
                     57225
                    8175
                   65400
                   ---------     A.
            4840 ) 6683.0625 ( 1.3807
                   ---------         4
                                ------
     A.  R.  P.                 1.5228
Anſwer 1 : 1 : 20                   40
                                -------
                                20.9120
                                -------
```

The Square Root of the Area of any Square gives the Length of its Side.

Example.

If the Area of a Court-yard that is ſquare, meaſure 6241 ſquare Feet; what is the Length of its Side?

```
         6241 ( 79 Feet the Length of its Side.
         49
         ----
    149) 1341
         1341
         ----
            0
            -
```

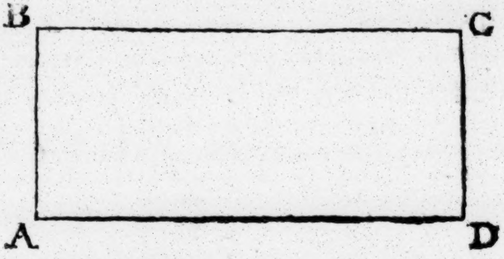
II. *To meaſure a Parallelogram.*

RULE.

Multiply the Length by the Breadth, the Product is the Content.

Examples.

Let A B C D repreſent a Parallelogram, whoſe Length A D is 43 Feet, and Breadth A B 22 Feet, requir-ed the Content in Yards?



43 Length.
22 Breadth.

86
86

9) 946 Area in Feet.

Yds. 105 : 1 *Foot. Anſwer.*

If the Length of a rectangular Graſs Plat meaſure 479 Yards, and its Breadth 184 Yards, how many Acres doth it contain?

479
184

1916
3832
479

A. *R.* *P.*
4840) 88136 (18 : 0 : 33 *Anſwer.*
4840

39736
38720

30.25) 1016 (33

N. B. $30\frac{1}{4}$ Yards make one Perch ſquare meaſure.

When the Area of any Parallelogram is divided by the $\left\{\begin{matrix}\text{Length}\\ \text{Breadth}\end{matrix}\right\}$ the Quotient is the $\left\{\begin{matrix}\text{Breadth}\\ \text{Length}\end{matrix}\right\}$ reſpectively.

If the ſuperficial Content of a Parallelogram be 88136 Yards, and 184 Yards broad, how long is it ?

```
184 ) 88136 ( 479 Length.
      736
      ----
      1453
      1288
      -----
       1656
       1656
       ----
```

Again, Let the Area be 88136, and Length 479, how broad is it ?

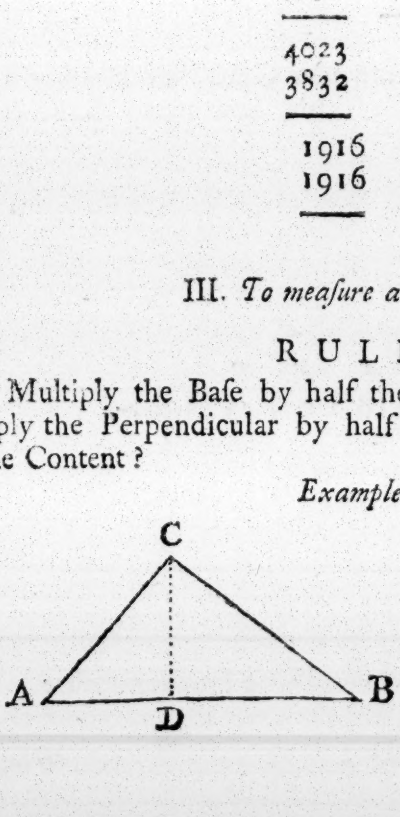
```
479 ) 88136 ( 184 Breadth.
      479
      ----
      4023
      3832
      ----
       1916
       1916
       ----
```

III. *To meaſure a Triangle.*

R U L E.

Multiply the Baſe by half the Perpendicular, or multiply the Perpendicular by half the Baſe, the Product is the Content ?

Examples.



Let the annexed Figure denote a Triangle whoſe Baſe A B is 24, and Perpendicular C D is 14, what is the Content ?

14 Perpend.	24 Baſe.	Or,	14 Perpend.
—	7	24	12
7 its half.	——	—	——
	168 Area.	12	168 Area.

If the Baſe of a Triangle meaſure 372 Feet, and Perpendicular 136 Feet, how many ſquare Yards doth it contain ?

```
2 ) 136          372
    ———           68
     68          ———
                 2976
                2232
                ————
              9) 25296 (
                ————
         Yds. 2810 : 6 Feet. Anſwer.
```

When the Area of any Triangle is divided by half the Baſe gives the Perpendicular, or by half the Perpendicular gives the Baſe.

Example.

Let the Baſe be 24, and Area 168, what is the Perpendicular ?

```
2 ) 24 (        12 ) 168 (
    —               ———
    12               14 Anſwer.
                     —
```

IV. *To find the Area of a Trapezium.*

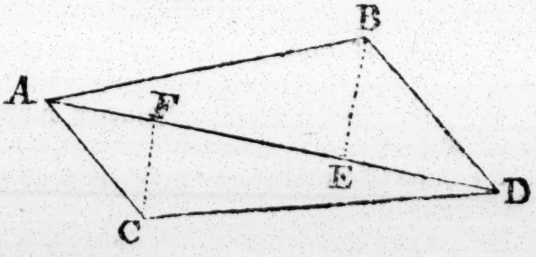
RULE.

Multiply the diagonal Line by half the Sum of the two Perpendiculars, the Product is the Area.

Or, Multiply the Sum of the two Perpendiculars by half the diagonal Line, the Product is the Area

Example.

Let ABDC denote a Trapezium, and its Diagonal AD meaſures 468, the Perpendicular BE 186, and CF 104, required the Area ?



BE = 186
CF = 104

Sum = 290

Half Sum = 145

Diagonal 468
145

2340
1872
468

67860 Area.

Or thus,

Diag. = 468

Half = 234

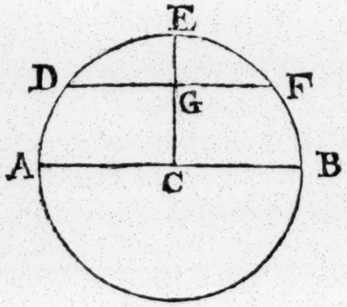
290 Sum Perpend.
234

1160
870
580

67860 Area.

V. *To meaſure a Circle.*

DEFINITIONS.



A B is the Diameter.
A C Radius.
D F Chord Line.
E G Verſed Sine.

RULES.

I. Diameter multiplied by 3.1416, gives the Circumference.

II. The Square of the Diameter multiplied by .7854, gives the Area.

III. The Circumference multiplied by .31831, gives the Diameter.

IV. The Square of the Circumference multiplied by .0795776, gives the Area.

V. Multiply the Square Root of the Area by 1.12837, gives the Diameter.

VI. Multiply the Square Root of the Area by 3.5449, gives the Circumference.

Examples.

I. The Diameter of a circular Graſs Plat was 35 Feet, how many Feet is it round ?

3.1416
35

157080
94248

109.9560 *Anſwer.*

II. If the Diameter of a Circle be 35, what is the Area?

35
35

175
105

1225 Square Diameter.
.7854

4900
6125
9800
8575

962.1150 Area.

III. If the Circumference be 144, what is the Diameter?

.31831
144

127324
127324
31831

45.83664 Diameter.

IV. If the Circumference be 1000, what is the Area?

1000	.079577
1000	1000000
1000000	79577.000000 Area.

V. If the Area of a Circle be 4840, what is the Diameter?

The Square Root of 4840 is 69.57.

Then 1.12837
by 69.57

789859
564185
1015533
677022

78.5007009 Diameter.

VI. If the Area of a Circle be 4840, what is the Circumference?

The Square Root of the Area 4840 is 69.57.

Then 3.5449
by 69.57

248143
177245
319041
212694

246.618693 Circumference.

VII. It is required to determine whether the Square or Circle has the greateſt Fencing round any given Piece of Land?

For Inſtance, what Difference is there in Fencing round an Acre of Land in a Square and in a Circle?

The Side of a Square of an Acre of Land is 69.57 for $\sqrt{4840} = 69.57$.

Then 4 times 69.57 = 278.28
Circumference for an Acre is = 246.618693 found above.

31.661307 Yards differ.

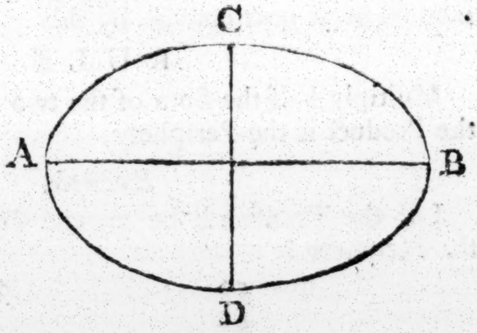
So the Square has 31 Yards and little more than half a Yard, Fencing than the Circle.

VII. *To find the Area of an Ellipſis or Oval.*

RULE.

Multiply the Tranverſe by the Conjugate Diameter, and that Product by .7854 gives the Area.

Example.



Let A B the tranſverſe Diameter be 50, and C D the Conjugate be 40, required the Area?

50
40
2000

.7854
2000
1570.8000 Area.

VIII. *To find the Periphery of an Ellipſis.*

RULE.

To twice the Square Root of the Sum of the Squares of the two Diameters, add one third Part of the Conjugate Diameter, and the Sum will be the Periphery.

Example.

Let tranſverſe A B = 50, Conjugate Diameter C D = 40, to find the Periphery?

50	40
50	40
2500	1600
1600	

Sum 4100 (64.0312 the Root.
2
128.0624
13.3333 being ⅓ of 40.
141.3957 the Periphery.

Or you may find the Periphery of any Ellipſis for common Buſineſs near enough by this

RULE.

Multiply half the Sum of the two Diameters by 3.1416 the Product is the Periphery.

Example.

Let the Tranſverſe be 50 and Conjugate 40, required the Periphery?

50	3.1416
40	45
Sum 90	157080
	125664
Half Sum 45	
	141.3720 the Periphery.

And this is near enough for any Practice.

SOLIDS.

A Cube is a Solid reſembling a Die, and is no more than a ſolid Square, to find the Solidity this is the

RULE.

Cube the Side, the laſt Product is the Solidity.

Example.

There is a Cube whoſe Side is 14 Inches, what is the Solidity ?

```
      14
      14
      ——
      56
     14
    ————
     196
      14
    ————
     784
    196
    ————
    2744 Solid Inches.
    ————
```

If you have the Area of any Cubic Veſſel, and would know the Dimenſions of it, extract the Cube Root of the Area, the Root is the Side.

Example.

I have a Cubical Veſſel whoſe ſolid Content is 2744 ſolid Inches, required the Side of it ?

```
                    2̇744̇ ( 14 Inches the Side.
                    1
                   ———
1×1=1×3=3 ) 17 ( 4
                   ———
   14 Cubed is 2744
                   ————
                      0
                     —
```

To find the Content of a Parallelopipedon.

A Parallelopipedon is a Solid Parallelogram, and to find the Content this is the

RULE.

Multiply the Length by the Breadth, and that Product again by the Depth, this laſt Product is the Solidity.

Example.

There is a Parallelopipedon, or a Block of Marble in that Shape, whoſe Length is 100 Inches, Breadth 20 Inches, and Depth or Thickneſs 14 Inches, required the Solidity ?

100
20

2000
14

28000 Solidity in Inches.

To find the Content of a Cylinder.

A Cylinder is a round Solid like a Garden Rolling Stone, having its Diameter alike from End to End of it. To find the ſolid Content this is the

RULE.

Multiply the Area of one End by the Length, gives the Solidity.

Example.

20 .7854
20 400

Square of Diameter 400 314.1600 Area of the End.
60

18849.6000 Solidity.

To find the Solidity of a Globe or Sphere.

A Globe or Sphere is a round Solid like a Ball, to find the Solidity this is the

RULE.

Multiply the Cube of the Diameter by .5236 gives the Solidity.

Example.

If a Cannon Ball be 8 Inches in Diameter, what is the Solidity ?

8 .5236
8 512
64 10472
8 5236
26180
Cube Diameter 512
268.0832 Solidity in Inches.

Multiply the Cube of the Circumference by .016887 gives the Solidity.

Example.

The Circumference of a Cannon Ball was 16 Inches, what is the Solidity?

16 .016887
16 4096
96 101322
16 151983
675480
256
16 69.169152 Solidity in Inches.
1536
256
4096

Of the Strength of Beams of Timber.

Mr *Emerſon*, at Page 117 of his *Principles of Mechanics*, treats of the Strength of Beams of Timber in all Poſitions; and their Streſs, by any Weights acting upon them, or by any Forces applied to them; where that excellent Author ſays, *The lateral Strength of any Piece of Timber, in any Place, whoſe Section is a Rectangle, is directly as the Breadth and Square of the Depth.*

Whence to find the Strength of any ſquare Piece of Timber this is the

RULE.

Multiply the Square of the Thickneſs in Inches, into its Breadth in Inches, and that again into the Weight that a Piece of the ſame ſort of an Inch ſquare and a Foot long would bear, and divide the Product by the Length of the Timber in Feet, gives the Anſwer.

Now to apply this Rule to Practice, Mr *Emerſon* has been pleaſed to give us ſome Experiments he made, with Pieces of Wood a Foot long and an Inch ſquare, ſupported horizontally at both Ends, what they will bear in the middle before they break, as follows, ſee Page 142.

Oak	—	320 Pounds.
Elm	—	310
Beech	—	290
Firr	—	280

Example.

What Weight will a Joiſt of Oak ſuſtain that is 12 Feet long, 8 Inches thick, and 6 Inches wide?

8
8
—
64 Square of the Thickneſs.
6
—
384
320
—
7680
1152
—
12) 122880
—
10240 *lb.* or 91*C.* 1*Qr.* 20*lb.* the Weight the Joiſt will bear, being hung in the middle.

But if the Joiſt be laid flat, then the Side 6 Inches is the Thickneſs, and 8 Inches the Breadth, and this is the Strength.

6

```
       6
       6
      ---
      36
       8
    ------
     288
      320
    ------
     576
    864
   -------
12) 92160
   -------
    7680
```

7680 *lb.* or 68 *C.* 2 *Qrs.* 8 *lb* the Joiſt will bear, when hung in the middle.

THE COMPUTATION OF BALLS and SHELLS.

PROBLEM I.

Given the Diameter of an Iron Bullet, to find its Weight.

RULE.

AS 100 is to the Cube of the Bullet's Diameter in Inches, ſo is 14 to the Weight in Pounds.

Example I.

If the Diameter of an Iron Bullet be 5 Inches, what is the Weight?

5
5
—
25
5
—
125 Cube of 5.

As 100 : 125 :: 14
14
—
500
125
—
1,00) 17,50
—
$17\frac{1}{2}$ *lb.* the Weight.

Example II.

If the Diameter of an Iron Bullet be $3\frac{1}{2}$ Inches, what is the Weight?

The Cube of 3.5 Inches = 42.875
As 100 : 42.875 : : 14
14

171500
42875

1,00)6,00.250 the Weight 6 *lb.*

But if the Bullet be Lead, to find its Weight, when the Diameter is known, this is the

RULE.

Cube the Diameter and multiply this Cube by .5236, and this laſt Product by .41, gives the Weight?

Example I.

What is the Weight of a Leaden Bullet, whoſe Diameter is 5 Inches?

Multiply .5236
By Cube of 5 = 125

26180
10472
5236

65.4500
.41

6545
26180

lb. 26.834500 the Weight.

Example II.

If a Leaden Bullet be 3 Inches Diameter, what is the Weight?

Multiply .5236
By Cube of 3 = 27

36652
10472

14.1372
.41

141372
565488

lb. 5.796252 the Weight.

PROBLEM II.

The Weight of an Iron Bullet being given, to find its Diameter.

RULE.

As 14 is to the given Weight, ſo is 100 to the Cube of its Diameter, the Cube Root thereof is the Diameter ſought.

Example I.

If an Iron Bullet weigh 6 Pounds, what is the Diameter?

lb.
As 14 : 6 : : 100
6

14) 600 (42.85 the Cube of the Diameter.
56

40 Its Log. is 1.63144
28

120 ⅓ of it is .54381
Its Number is 3.5 the Diameter.
112

80
70

10

Example

Example II.

lb.
As 14 : 17.5 : : 100
100

14) 1750.0 (125 the Cube of the Diameter.

Whoſe Cube Root is 5 Inches, the Diameter ſought.

PROBLEM III.

The Weight of a Leaden Bullet being given, to find its Diameter.

RULE.

Divide the given Weight by .41 (the ſpecific Gravity of Lead in Pounds) and that Quotient again by .5236, the laſt Quotient gives the Cube of the Diameter, whoſe Cube Root is the Diameter ſought.

Example.

If a Leaden Bullet weigh 26.8345 Pounds, what is the Diameter?

.41) 26.8345 (65.45
246
223
205
184
164
205
205

.5236) 65.45 (125 the Cube of the Diameter.
52 36
13090
10472
26180
26180

Its Log. 2.09691
A third Part .69897
Its Number 5 the Diameter ſought.

When Guns are fortified alike, that is, when they are in the ſame Proportion in Weight and Thickneſs, their requiſite Charges of Powder may be found by the following

PROBLEM IV.

Given the requiſite Charge of Powder for one Gun, to find the requiſite Charge of Powder for another Gun.

RULE.

As the Cube of the known Gun's Diameter at the Bore, is to its given Charge of Powder, ſo is the Cube of the Diameter of that Gun, whoſe Charge of Powder is required, to its Charge required.

Example.

The Diameter at the Bore of a nine Pounder is 4.3 Inches, and its Charge is 4½ Pound of Powder, what will a twenty four Pounder require, whoſe Diameter at the Bore is 5.9 Inches?

```
    4.3            5.9
    4.3            5.9
   ----           ----
    129            531
   172            295
   ----           ----
   18.49          34.81
     4.3            5.9
   -----          ------
    5547          31329
   7396          17405
  ------         -------
  79.507         205.379
  ------         -------
```

```
             lb.
As 79.507 : 4.5 :: 205.379
                       4.5
                  --------
                   1026895
                   821516
                  --------
      79.507 ) 924.2055 ( 11.6 lb. required.
               --------
```

Whence the following Table is eaſy to be underſtood by Inſpection.

A TABLE of GUNNERY.

Guns. Pounders	Diameter of the Bore.	Diameter of the Bullet.	Powder for Proof.		Powder for Service.	
			Braſs. Land Service.	Braſs and Iron. Sea Service.	Braſs. Land Service.	Braſs and Iron. Sea Service.
	Inches.	*Inches.*	*lb. oz.*	*lb. oz.*	*lb. oz.*	*lb. oz.*
½	1.7	1.5	0 : 8	0 : 8	0 : 4	0 : 4
3	2.9	2.7	3 : 0	3 : 0	1 : 8	1 : 8
6	3.6	3.4	6 : 0	6 : 0	3 : 0	3 : 0
9	4.2	4.	9 : 0	9 : 0	4 : 8	4 : 8
12	4.6	4.4	12 : 0	12 : 0	6 : 0	6 : 0
18	5.3	5.	18 : 0	15 : 0	9 : 0	9 : 0
24	5.9	5.5	21 : 0	18 : 0	12 : 0	11 : 0
32	6.4	6.1	26 : 0	21 : 8	16 : 0	14 : 0
42	7.	6.6	31 : 8	25 : 0	21 : 0	17 : 0

To find the proper Adjuſtment of Powder for any Piece of Ordnance, whether true bored or not, ſo as to render the Bullet capable of doing the greateſt Execution, is a very difficult Problem in Gunnery, and though ſeveral Rules have been given, Engineers have found them often fail: and this I take to lie in the different Strength of Powder, or in the Piece of Ordnance, or in both, therefore Experience will be your beſt and ſureſt Guide. Engineers uſually allowed two thirds of the Weight of the Ball, for its proper Charge; but Experience ſhewed that this was more than neceſſary, and then they allowed about one half its Weight, which continues pretty nearly the ſame to this Day. In *October* 1739, the *French* Engineers made ſeveral Experiments at *La Fere*, to adjuſt the Charge to the Piece, by which it appeared from the Reſult of the Experiments, ſays M. *Bigot de Morogues*,

That a 24 Pounder ought to be charged with no more than 9 Pounds of Powder.

16 Pounders with 6 Pounds.
12 Pounders with 5 Pounds.
8 Pounders with 3 Pounds.

And that larger Charges than theſe do not (ſays he) the leaſt augment the Flight of the Bullet.

But perhaps the Charge may vary between our Pieces and the *French*, as it is ſaid the Dimenſions of our Pieces do not correſpond with theirs: but it will be very eaſy to find the proper Charge, by firing it ſucceſſively with different Quantities of Powder, and obſerving which carries the Bullet to the greateſt Diſtance.

PROBLEM V.

To find the Solidity of any Bullet.

RULE.

Multiply the Cube of the Diameter of the Bullet in Inches by .5236, the Product is the Solidity in Inches.

Example.

If the Diameter of a Bullet is 6 Inches, what is the Solidity?

The Cube of 6 is = 216, then .5236
216

31416
5236
10472

113.0976 Solid Inches.

PROBLEM VI.

To find the Cubic Inches in any Bomb, and how much Powder will fill it.

RULE.

Multiply the Cube of the Diameter of the hollow Part of the Bomb by .5236 gives the Cubic Inches, and that Product again by .032, the last Product will give the Quantity of Powder required in Pounds.

Example.

A Bomb for a 13 Inch Mortar its hollow Part is 12$\frac{3}{4}$ Inches, its concave Axis being about 8$\frac{1}{4}$ Inches, it is required to know how much Powder will be sufficient to fill this Shell?

8.25
8.25

4125
1650
6600

68.0625
8.25

3403125
1361250
5445000

561.515625 the Cube of the hollow Part's Diameter.
.5236

3369093750
1684546875
1123031250
2807578125

294.0095812500

And 294

$$\begin{array}{r} 294 \\ .032 \\ \hline 588 \\ 882 \\ \hline 9.408 \end{array}$$

9.408 *lb.* of Powder to fill the Shell.

I ſhall take the Liberty to inſert here a Table founded on Experiment made in the Year 1743-4, by the ingenious Mathematician Mr *W. Mountaine*, F. R. S. Maſter of the Academy in *Gainsford-ſtreet*, *Southwark*, from Page 116 of his *Practical Sea Gunner's Companion*, which the Reader may compare with the above.

A Table of Quantities of Powder for filling Shells, with Weight of Compoſition requiſite to fill each Fuze, Length of the Fuze, and Number of Seconds each Fuze burnt, when cut to its Length for fixing in the Shell.

Experimented in the Year 1743·4.

Nature of Shells.	Quantity of Powder to fill each Shell.	Length and Diameter of Fuze Holes.		Weight of Fuzes when		Weight of Comp with No of Ladles to fill each Fuze.		Length of Solid Comp. when cut to fix in Shells.	Time of Fuzes burning.	Time of one Inch of Comp. burning.	Weight of Shells empty.
		Length	Diam	Empty	filled with Comp.	Comp.	No of Ladles.				
Inches.	*lb.* *oz.*	*In.* *Pts.*	*In.* *Pts.*	*oz.* *dr.*	*oz.* *dr.*	*oz.* *dr*		*In.* *Pts.*	*Seconds.*	*Seconds.*	*lb.* *oz.*
$12\frac{3}{4}$	9 : $4\frac{1}{2}$	8 . 4	0 . 6	9 : 4	11 : 10	2 : 6	38	8 . 2	37	$4\frac{3}{4}$ *ferè*	196 : 10
$9\frac{3}{4}$	4 : 4	7 . 3	0 . 5	5 : 12	7 : 3	1 : 7	38	7 . 2	31	4	96 : $6\frac{1}{2}$
$7\frac{3}{4}$	2 : $3\frac{1}{2}$	6 . 6	0 . 44	3 : 12	4 : 11	0 : 15	36	6 . 2	30	$4\frac{3}{4}$	46 : 12
$5\frac{1}{2}$	1 : $1\frac{1}{2}$	3 . 7	0 . 20	1 : 4	1 : 8	0 : 4	23	3 . 6	16	$4\frac{1}{2}$ *ferè*	14 : $4\frac{1}{2}$
$4\frac{2}{5}$	0 : 8	3 . 4	0 . 20	1 : 1	1 : 4	0 : 3	21	3 . 1	15	$4\frac{3}{4}$	7 : $12\frac{1}{2}$
Hand Granad. Sea	0 : $4\frac{1}{2}$	2 . 5	0 . 20	0 : 7	0 : 9	0 : 2	19	2 . 2	$10\frac{1}{2}$	$4\frac{1}{2}$	4 : 6
Ditto Land	0 : 2										1 : 11.1
Muſquet	0 : $1\frac{1}{16}$										1 : 3.14

The ſollowing Table is in Mr *Stone*'s Mathematical Dictionary, and is Mr *Anderſon*'s, ſhewing the requiſite Weight of Powder for all Mortars, from 6 to 20 Inches Diameter

Diameter in Inches.	Weight. Pounds.	Weight. Ounces	Diameter in Inches.	Weight. Pounds.	Weight. Ounces.
6	0 :	13	13½	9 :	10
6½	1 :	1	14	10 :	11½
7	1 :	5	14½	11 :	14
7½	1 :	10	15	13 :	3
8	2 :	0	15½	14 :	9
8½	2 :	6	16	16 :	0
9	2 :	14	16½	17 :	9
9½	3 :	6	17	19 :	3
10	3 :	14½	17½	20 :	15
10½	4 :	8	18	22 :	12½
11	5 :	3	18½	24 :	11
11½	5 :	15	19	26 :	13
12	6 :	12	19½	28 :	14
12½	7 :	10	20	31 :	4
13	8 :	9			

For Proof, the Chamber of the Mortar is moſtly filled full of Powder; but for Service, the Quantity is diminiſhed, or increaſed, as the Random is leſs or greater.

⁂

Some of the moſt uſeful THEOREMS *in plane* Geometry *demonſtrated.*

IT will be proper to premiſe firſt the following *Axioms*, being ſelf-evident Principles.

I. Things that are equal to one and the ſame Thing, are equal to one another.

II. If equal Things be added to equal Things, the Sum will be equal.

III. If equal Things be ſubtracted from equal Things, the Remainder will be equal.

IV. If equal Things be multiplied by equal Things, the Product will be equal.

V. If equal Things be divided by equal Things, the Quotient will be equal.

VI. Things which being laid upon one another do agree, or meet in all their Parts, are equal to one another.

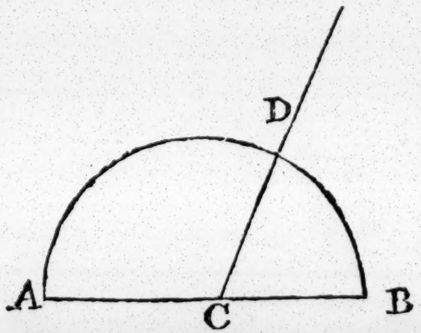
VII. Every Whole is equal to all its Parts taken together; and therefore every Whole is greater than its Part.

THEOREM I.

If a right Line ſtands upon another right Line, the Angles which it makes with it will either be two right Angles, or together equal to two right Angles.

DEMONSTRATION.

Let the Lines be A B and C D meeting in the Point C, upon which Point deſcribe any Circle at Pleaſure.



Then the Arch A D is the Meaſure of the $\angle$ A C D, and the Arch D B of the $\angle$ D C B; but the Arches A D $+$ D B $=$ 180 Degrees, ſince they complete a Semicircle;

conſequently $\angle ACD + DCB = 180^\circ =$ to two right Angles. *Q. E. D.*

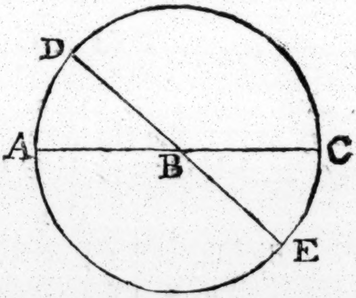
Corollary 1. Hence it follows, that if $\angle ACD = 90^\circ$, then muſt $\angle DCB = 90^\circ$; but if $\angle ACD$ be obtuſe, then the $\angle DCB$ will be acute.

Cor. 2. And if ever ſo many right Lines ſtand upon a right Line at one and the ſame Point, all the Angles taken together will be equal to two right Angles.

THEOREM II.

If two Lines cut or croſs each other, the two oppoſite Angles will be equal.

DEMONSTRATION.



Let the two right Lines be A C, D E, croſſing each other in B.

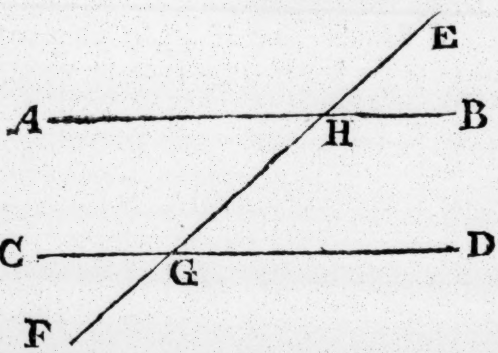
Theo. 1.	1	$\angle ABD + \angle DBC = \angle DBC + CBE = 180^\circ$
$1 - \angle DBC$	2	$\angle ABD = \angle CBE$ by Ax. 1.
And	3	$\angle DBC = \angle ABE$ by the ſame.

Cor. Hence it is evident that if two or more right Lines interſect each other, in one and the ſame Point, they will make twice as many Angles as there are Lines; and all the Angles that are capable of being found about one Point, are equal to a whole Circle, or 360 Degrees.

THEOREM III.

Any right Line croſſing two Parallels makes the alternate Angles on the ſame Side equal to one another, *i. e.* $\angle AHF = \angle CGF$.

DEMONSTRATION.



Let the two parallel right Lines AB, CD be croſſed by the Line EF.

Since parallel Lines are ſuch right Lines, in ſome plane, which if infinitely produced will never meet, and Th. parallel Lines muſt have the ſame Inclination to any right Line that croſſeth them, and conſequently $\angle AHF = \angle CGF$. *Q. E. D.*

THEOREM IV.

Any right Line croſſing two Parallels (*ſee the laſt Figure*) makes the alternate Angles on different Sides equal, *i. e.* $\angle AHG = \angle HGD$

DEMONSTRATION.

By Th. 3.	1	$\angle AHG = \angle CGF$.
Th. 2	2	$\angle CGF = \angle HGD$.
℘ Ax. 1. 1=2	3	$\angle AHG = \angle HGD$. *Q. E. D.*

THEOREM V.

If a right Line croſs two Parallels, the two internal Angles are together equal to two right Angles, *i. e.* $\angle AHG + \angle CGH = 180°$. *See laſt Figure.*

DEMONSTRATION.

By Th. 4.	1	$\angle AHG = \angle HGD$.
(Ax. 2.)+CGH	2	$\angle AHG + \angle CGH = \angle HGD + \angle CGH$
But	3	$\angle HGD + \angle CGH = 180°$, by Th. 1.
And Ax. 1.	4	$\angle AHG + \angle CGH = 180°$. *Q. E. D.*

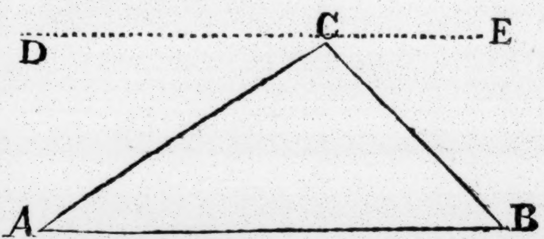
THEO-

THEOREM VI.

The three Angles of every plane Triangle are equal to two right Angles.

DEMONSTRATION.

Draw the right Line DE, through the Angular Point C, and parallel to the Side A B. Then



By Th. 4.	1	$\angle A = \angle ACD$.
And	2	$\angle B = \angle BCE$.
1+2	3	$\angle A + \angle B = \angle ACD + \angle BCE$.
3+$\angle ACB$	4	$\angle A + \angle B + \angle ACB = \angle ACD + \angle BCE + \angle ACB$.
The. 1. Cor 2.	5	$\angle ACD + \angle BCE + \angle ACB = 180°$.
Ax. 1.	6	$\angle A + \angle B + \angle ACB = 180° =$ two right Angles. *Q. E. D.*

Corollary Hence if the Sum of any two Angles be ſubtracted from 180°, the Remainder will be the other Angle; and if one Angle of a plane Triangle be 90°, the Sum of the other two acute Angles will be equal to 90°.

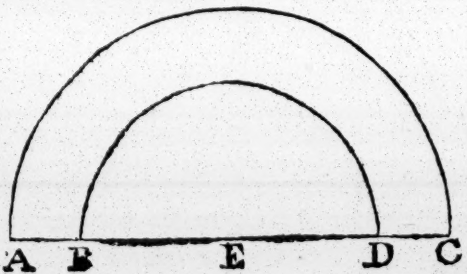
THEOREM VII.

The Sum and Difference of any two Quantities being given, the Quantities themſelves are thereby given.

For half the Sum added to half the Difference gives the greater, and half the Difference ſubtracted from half the Sum gives the leſſer Quantity.

DEMONSTRATION.

Let A B repreſent the leſſer, and B C the greater Quantity, make DC =AB, biſect BD in E, then will AE = EC = half the Sum, and BE = ED =



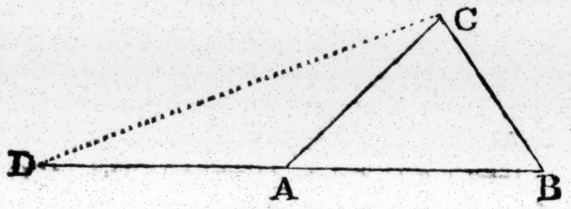
half the Difference; hence it is plain that $BE+EC=BC$, the greater; and $AE-BE=AB$, the leſſer Quantity.

Q. E. D.

THEOREM VIII.

If one Side of any plane Triangle be produced, the outward Angle will be equal to the two inward oppoſite ones.

DEMONSTRATION.

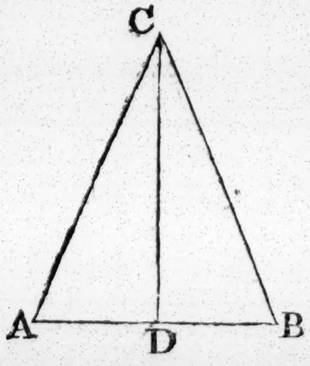


By Th. 1.	1	$\angle DAC+\angle CAB=180°$.
And Th. 6.	2	$\angle B+\angle C+\angle CAB=180°$.
$1=2$	3	$\angle DAC+\angle CAB=\angle B+\angle C+\angle CAB$
$3-\angle CAB$	4	$\angle DAC=\angle B+\angle C$. *Q. E. D.*

THEOREM IX.

In every plane Triangle, equal Sides ſubtend or are oppoſite to equal Angles; and conſequently equal Angles in the ſame Triangle are ſubtended by equal Sides.

DEMONSTRATION.



Let the Triangle ABC be Iſoſceles, that is, let $AC=BC$, then I ſay $\angle A=\angle B$; for let AB be biſected in D, and CD joined; then ſince $AC=BC$, $AD=DB$, and CD common; if the Triangle CDB be placed upon ACD, ſo that CB lie upon its equal AC, then muſt the Side DB fall upon its equal AD, and the Line CD upon itſelf being common; and Th the Triangle CBD will coincide in all its Parts with the Triangle ADC, hence by Ax. 6. $\angle A=\angle B$.

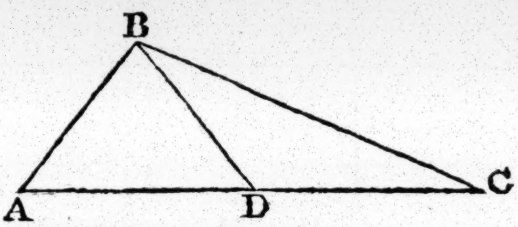
Corollary. Hence it follows that the three Angles of every equilateral Triangle are equal to one another, and therefore each equal to 60 Degrees.

THEOREM X.

In every Triangle the greateſt Side ſubtends the greateſt Angle; that is, if AC be greater than AB, then will $\angle$ABC be greater than the $\angle$BCA.

DEMONSTRATION.

Make AD= AB, and join BD. Then



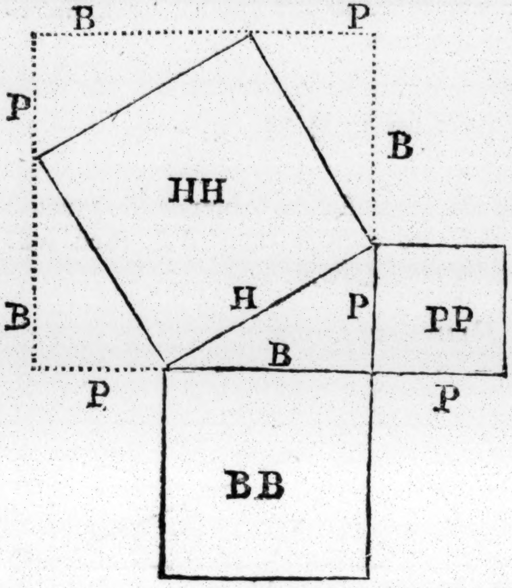
The. 9.	1	$\angle ABD = \angle ADB$, and
The. 8	2	$\angle C + \angle DBC = \angle ADB$.
1=2	3	$\angle C + \angle DBC = \angle ABD$, by Ax. 1.
But	4	$\angle$ABC is greater than its Part ABD, by Ax. 7.
Therefore	5	$\angle C + \angle DBC$ muſt be leſs than $\angle$ABC and
Conſequently	6	$\angle$C is leſs than $\angle$ABC.

Q. E. D.

THEOREM XI.

In any right angled Triangle BPH, the Square HH, deſcribed upon the Hypotenuſe H, is equal to the Sum of the Squares BB, PP, deſcribed on the two other Sides B, P.

DEMONSTRATION.



Make a Square whoſe Side is equal to $B+P$, and let that Square be repreſented by S, ſo will its Area be $BB+2BP+PP$. Alſo the Sum of the Areas of the four Triangles included in that Square will by Menſuration be $\frac{1}{2}BP\times4=2BP$.

I ſay $HH=BB+PP$.

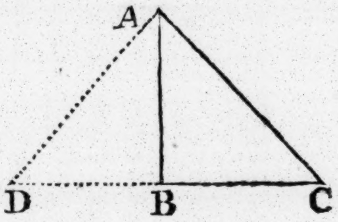
For	1	$S=HH+2BP$.
And	2	$S=BB+2BP+PP$.
$1=2$	3	$HH+2BP=BB+2BP+PP$.
$3-2BP$	4	$HH=BB+PP$.

Corollary.

	1	$H=\sqrt{BB+PP}$.
For	2	$HH=BB+PP$.
2ɯ2	3	$H=\sqrt{BB+PP}$.
	1	$B=\sqrt{HH-PP}$.
For	2	$BB+PP=HH$.
$2-PP$	3	$BB=HH-PP$.
3ɯ2	4	$B=\sqrt{HH-PP}$.
	1	$P=\sqrt{HH-BB}$.
For	2	$PP+BB=HH$.
$2-BB$	3	$PP=HH-BB$.
3ɯ2	4	$P=\sqrt{HH-BB}$.

Or this DEMONSTRATION.

Let ABC be the right angled Triangle, make $\angle CAD = 90^\circ$, and draw AD to meet CB produced in D, then by



Theo. 17. following $BC : BA :: BA : \frac{BA^2}{BC} = DB$.

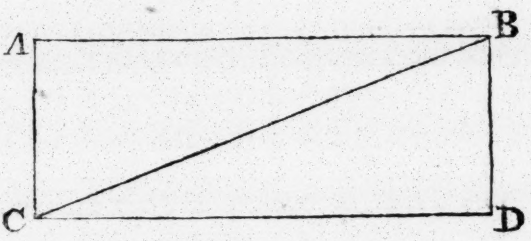
Then will $DC = \frac{BA^2 + BC^2}{BC} : AC :: AC : BC$, and multiplying Extremes and Means we have $AC^2 = AB^2 + BC^2$.
Q. E. D.

THEOREM XII.

The Diagonal of a Parallelogram divides it into two equal Triangles.

DEMONSTRATION.

Since AC, BD, and AB, CD, are Parallels.

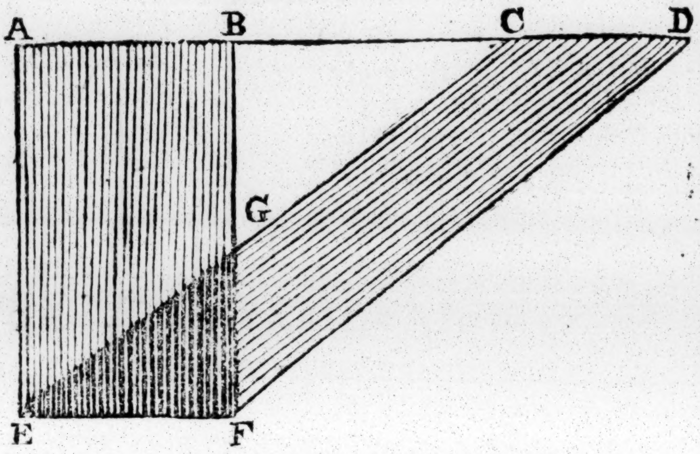


By Theo. 4. | 1 | $\angle ACB = \angle CBD$.
And | 2 | $\angle BCD = \angle ABC$.
Therefore | 3 | $\triangle ABC = \triangle CBD$, being equiangular Triangles, and AC common.
Q. E. D.

THEOREM XIII.

Parallelograms having the ſame Baſe and Altitude, are equal.

DEMONSTRATION.



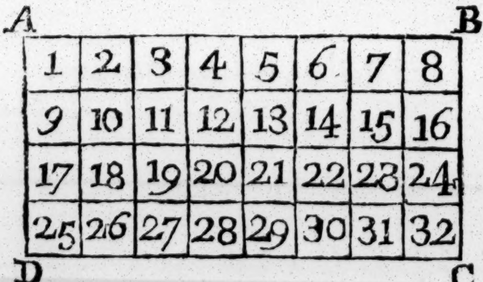
Let the two Parallelograms ABFE, CDEF, ſtand upon the ſame Baſe E F, and between the ſame Parallels E F, A D.

Since | 1 | AB$=$EF$=$CD by Conſtruction.
Theref. | 2 | AC$=$BD, and
Becauſe | 3 | AE$=$BF.
And | 4 | $\angle$A$=$$\angle$B.
Then | 5 | $\triangle$ACE$=$$\triangle$BFD, take the Triangle BCG from both, and there remains the Trapezia ABGE$=$CDFG add the Triangle EFG to both Sides and the Sum is

▭ A B F G $=$ ▭ C D F E. *Q.E.D.*

Corollary. Since the Diagonal of every Parallelogram divides it into two equal Triangles by *Theorem* 12. Therefore Triangles conſtituted upon the ſame Baſe, and between the ſame Parallels, are equal.

Lemma. A right Line is ſaid to be multiplied by another right Line, when a Parallelogram is made of the two Lines; that is, the Product of the two Numbers which expreſs the equal Parts in the two Sides, are equal to what the Mathematicians call the ſuperficial Content or Area of ſuch Figures,



1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32

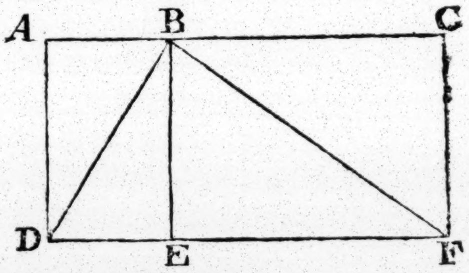
Figures, or they are underſtood to contain ſo many little Squares, whoſe Sides are each equal to one of the equal Parts with which the Sides were meaſured. Thus for inſtance, let $AB = CD = 8$, $AD = BC = 4$, then $AD \times AB = 4 \times 8 = 32$, which is the Area or Number of little Squares contained in the Parallelogram.

THEOREM XIV.

Parallelograms and alſo Triangles, that ſtand between the ſame Parallels, or which have the ſame Height, are in a direct Proportion to one another as their Baſes.

DEMONSTRATION.

By the above *Lemma.*



$$\text{The Area of the } ▭ \begin{cases} BD = BE \times DE. \\ BF = BE \times EF. \end{cases}$$

which Areas are in the Proportion of $DE : EF$,

For $BE \times DE : BE \times EF :: DE : EF$.

That is, ▭ BD : ▭ $BF :: DE : EF$.

$$\text{Again by Theo. 12. Area } \triangle \begin{cases} DBE = \tfrac{1}{2} BE \times DE. \\ EBF = \tfrac{1}{2} BE \times EF. \end{cases}$$

Then will $\triangle DBE : \triangle EBF :: DE : EF$.

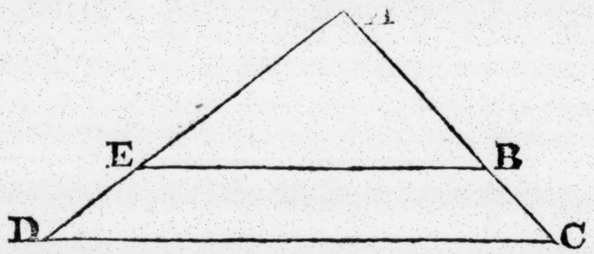
Q. E. D.

THEOREM XV.

If a right Line be drawn parallel to any Side of a plane Triangle it will cut from the given Triangle another leſſer Triangle, that will be ſimilar or equiangled to the whole Triangle.

DEMONSTRATION.

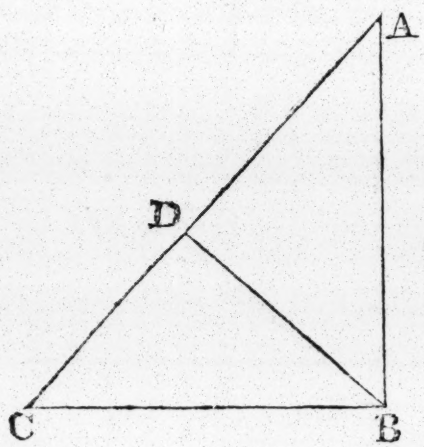
By Theo. 3. $\angle B = \angle C$, $\angle E = \angle D$, and ſince $\angle A$ is common, it is evident that the Triangle ABE is ſimilar or equiangled to the Triangle ACD. *Q. E. D.*



THEOREM XVI.

If a Perpendicular be let fall from the right Angle of a right angled Triangle upon the Hypotenuſe, it will divide the Triangle into two right angled ones, ſimilar to the Whole and to each other.

DEMONSTRATION.



Draw the Lines as ⅌ Figure.

Where	1	$\angle CDB = \angle BDA = \angle CBA = 90^\circ$.
Therefore	2	$\angle CBD + \angle DBA = 90^\circ$.
And	3	$\angle C + \angle CBD = 90^\circ$ by Cor. 1. The. 6.
$2 = 3$	4	$\angle CBD + \angle DBA = \angle C + \angle CBD$.
$4 - \angle CBD$	5	$\angle DBA = \angle C$.
Again	6	$\angle A + \angle ABD = \angle ABD + \angle DBC = 90^\circ$
$6 - \angle ABD$	7	$\angle A = \angle DBC$.

Therefore the Triangles CDB, BDA, CBA are ſimilar. *Q. E. D.*

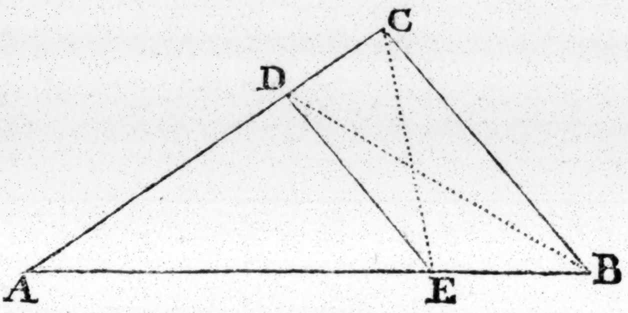
THE-

THEOREM XVII.

If two Triangles be ſimilar, their like Sides, or the Sides ſubtending equal Angles will be proportional.

DEMONSTRATION.

Let DE be parallel to CB, then will the Triangles ABC, AED be equiangular, join DB and CE, then by Cor. to



Theo. 13. 1 $\triangle ECD = \triangle DBE$.
By Theo. 14. 2 $\triangle ADE : \triangle ABD :: AE : AB$.
And 3 $\triangle ADE : \triangle ACE :: AD : AC$.

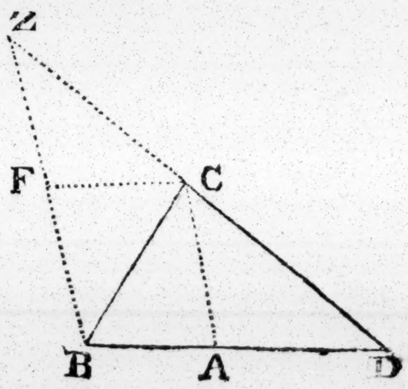
Since the $\triangle DBE = \triangle ECD$, therefore $\triangle ABD = \triangle ACE$. Hence $AE : AB :: AD : AC$. *Q. E. D.*

THEOREM XVIII.

If any Angle of a plane Triangle be biſected by a right Line, it will cut the oppoſite Side in proportion to the other two Sides of the Triangle.

DEMONSTRATION.

Produce the Side DC till $CZ = CB$, join ZB, and draw CF parallel to BD, then will the Triangles CFZ, DCA be ſimilar, for



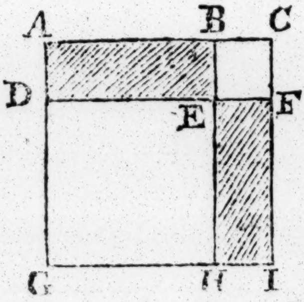
By Theo. 3.	1	$\angle ZCF=\angle D$.
And Th. 8.	2	$\angle Z+\angle ZBC=\angle BCD$.
But	3	$\angle Z=\angle ZBC$.
Becauſe	4	$ZC=BC$.
Therefore	5	$\angle Z=\angle ACD$.
And	6	$\angle A=\angle F$.
Since	7	$\angle BCA=\angle FBC$, the Lines CA, ZB, are parallel, and
Thence	8	$BA=FC$, and
Theo. 17.	9	$BA\,(=FC) : BC\,(=ZC) :: AD : CD$ or which is the
Same	10	$AB : AD :: BC : CD$.

Q. E. D.

THEOREM XIX.

If a right Line be divided into two Parts or Segments, the Square of the whole Line will be equal to the Square of each Segment, together with a double Rectangle of the ſame Segments.

Demonstration.



Let CG be the Square of AC and GE that of AB, and let HE, DE be produced to meet the Sides of the Square GC in B and F, then ſince $DE=EH$, EC will be the Square of BC, and $AE=FH=$ a rectangle under AB, BC, but $CG=GE+EC+AE+FH$; that is, $\overline{AC}^{2}=\overline{AB}^{2}+\overline{BC}^{2}+\overline{2AB\times BC}$.

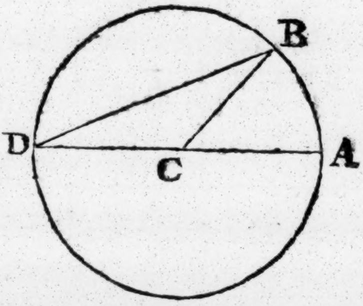
Q. E. D.

THEOREM XX.

The Angle at the Center of any Circle is double to the Angle at the Periphery, when both the Angles ſtand upon the ſame Arch.

DEMONSTRATION.

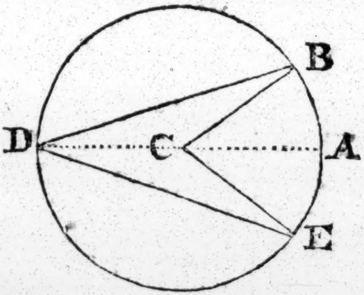
Caſe I.



Let the Diameter AD and Line BD form the Angle at the Periphery, draw the Radius CB, then I ſay $\angle ACB$ is double $\angle D$.

For | 1 | $\angle BCA=\angle D+\angle DBC$, by Th. 8.
But | 2 | $\angle D=\angle DBC$, by Th 9 becauſe $DC=CB$.
Theref. | 3 | $\angle BCA=2\angle D$.

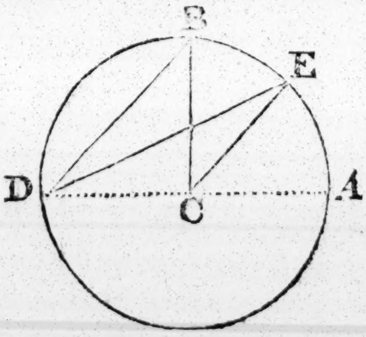
Caſe II.



Draw the Diameter AD, then by

Caſe I. | 1 | $\angle ACB=2\angle ADB$.
And | 2 | $\angle ACE=2\angle ADE$.
1+2 | 3 | $\angle ACB+\angle ACE=2\angle ADB+2\angle ADE$, by Ax. 2.
That is, | 4 | $\angle ECB=2\angle EDB$.

Caſe III.



Draw the Diameter AD, then by

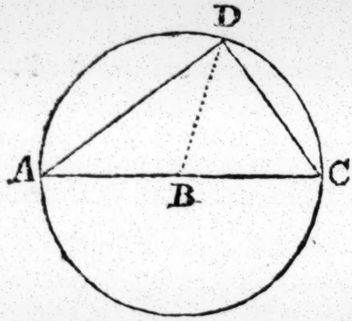
Caſe I 1 $\angle ACB=2\angle ADB$.
And 2 $\angle ACE=2\angle ADE$.
1—2 3 $\angle ACB-\angle ACE=2\angle ADB-2ADE$.
That is, 4 $\angle ECB=2\angle EDB$. *Q. E. D.*

Corollary. Hence all the Angles at the Periphery that ſtand upon the ſame or equal Arches are equal to one another.

THEOREM XXI.

The Angle in a Semicircle is a right Angle.

DEMONSTRATION.



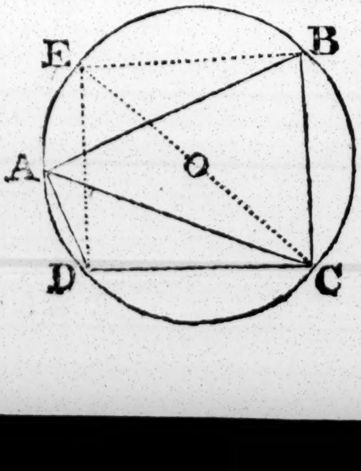
Let ABC be the Diameter, and draw the Radius BD, then by

Th. 9. 1 $\angle A=\angle ADB$.
And 2 $\angle C=\angle CDB$.
1+2 3 $\angle A+\angle C=\angle ADB+\angle CDB$, by Ax. 2.
That is 4 $\angle A+\angle C=\angle ADC$.
But 5 $\angle A+\angle C+\angle ADC=180^\circ$, and
Theref. 6 $\angle ADC=90^\circ$. *Q.E.D.*

THEOREM XXII.

An Angle in a Segment leſs than a Semicircle is greater than a right Angle, and an Angle in a Segment greater than a Semicircle is leſs than a right Angle.

DEMONSTRATION.

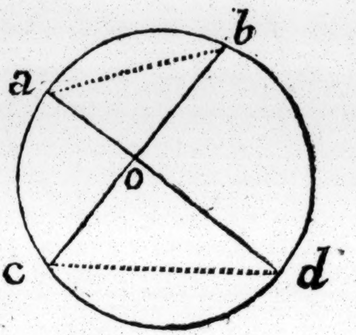


Let the $\angle ABC$ be an Angle in the Segment AEBC greater than a Semicircle; and $\angle ADE$ an Angle in the Segment CDA leſs than a Semicircle. Draw the Diameter CE, and join BE, ED, then by Theor. 21. $\angle EDC =90^\circ=\angle EBC$, it is thence evident that $\angle$ADC is greater than $\angle$EDC, and alſo that $\angle$ABC is leſs than $\angle$EBC. *Q. E. D.*

THEOREM XXIII.

If two Lines be any how drawn within a Circle cutting each other, the Rectangle of the Segments of one Line will be equal to the Rectangle of the Segments of the other Line.

DEMONSTRATION.

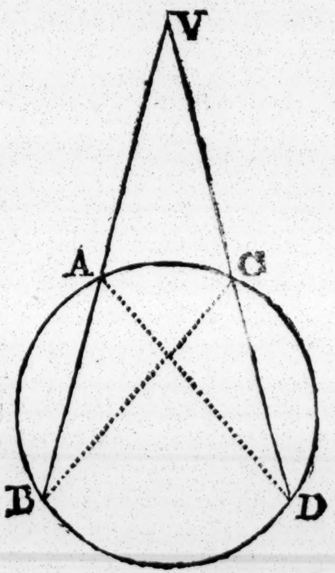


Join the Points *ab*, and *cd*, then will the Triangles *aob*, and *cod* be ſimilar, for $\angle a = \angle c$, and $\angle b = \angle d$, by Cor. to The. 20, therefore by Th. 17, $oc : od :: ao : ob$. And by multiplying Extremes and Means we have $co \times ob = ao \times od$.

Q. E. D.

THEOREM XXIV.

If from a Point without a Circle two right Lines be drawn, to the oppoſite Part of the Periphery; the Rectangle of one whole Line and its Part without the Circle, is equal to the Rectangle of the other whole Line, and its Part without the Circle.



DEMONSTRATION.

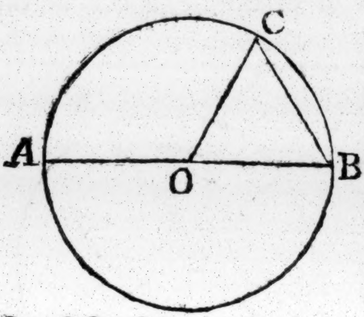
Draw the Lines BC, AD, then the Triangles VBC, VAD are ſimilar, for $\angle D = \angle B$ by Cor. to Theo. 20. and $\angle V$ common, hence $VB : VC :: VD : VA$, and multiplying Extremes and Means $VB \times VA = VC \times VD$.

Q. E. D.

THEOREM XXV.

The Chord of 60° of the Arch of any Circle is equal to the Radius or Semidiameter of that Circle.

DEMONSTRATION.



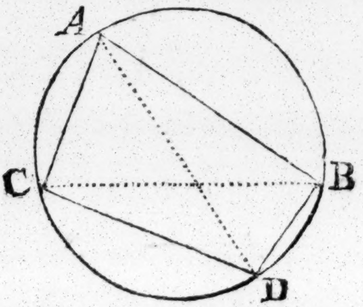
Let BC be an Arch of 60°, BC its Chord, then $\angle COB = 60°$, and $\angle AOC = 120° = \angle C + \angle B$ by Theor. 8. but ſince $OB = OC$, the $\angle C = \angle B$, and therefore each $= 60°$, Th. ſince all the Angles are equal the Sides muſt be equal by Theo. 9. that is, $BO = OC = BC$ the Radius.

Corollary. Hence the Reaſon is evident why 60 is taken from the Line of Chords on any Scale for the Radius of a Circle ſuited to that Scale.

THEOREM XXVI.

If any Trapezium be inſcribed in a Circle the two oppoſite Angles taken together are equal to two right Angles.

DEMONSTRATION.



Draw the Diagonals, AD CB.

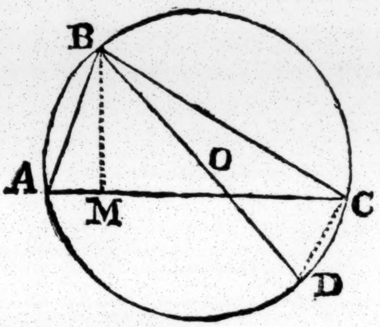
Then | 1 | $\angle ADC = \angle ABC$ } by Cor. to Th 20.
And | 2 | $\angle ACB = \angle ADB$ } by Cor. to Th 20.
But | 3 | $\angle CAB + \angle ACB + \angle CBA = 180°$ by T. 6.
And | 4 | $\angle CDA + \angle ADB = \angle CDB$.
Thereſore | 5 | $\angle CAB + \angle CDB = 180°$.
And ſo is, | 6 | $\angle ACD + \angle ABD = 180°$.

Q. E. D.

THEOREM XXVII.

If from any Angle of a plane Triangle inſcribed in a Circle, there be let fall a Perpendicular upon the oppoſite Side, as that Perpendicular is in proportion to one of the Sides including the Angle, ſo is the other Side including the Angle to the Diameter of the Circle.

DEMONSTRATION.



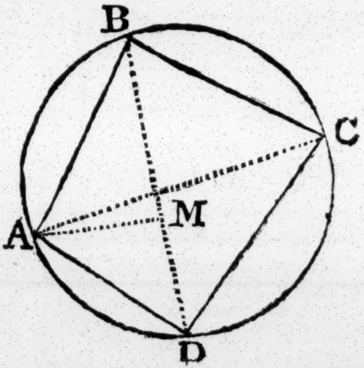
Draw the Diameter BD, and join CD, then the Triangles BAM, BCD, are ſimilar.

For | 1 | $\angle A = \angle D$, by Cor. Th. 20.
And | 2 | $\angle M = \angle BCD = 90°$. Th. 21.
Hence | 3 | BM : AB : : BC : BD, Th. 11. *Q. E. D.*

THEOREM XXVIII.

If any Trapezium be inſcribed in a Circle, there be drawn two Diagonals, their Rectangle is equal, to the Sum of the Rectangles under the oppoſite Sides of the Trapezium.

DEMONSTRATION.



Make $\angle DAM = \angle CAB$, then the Triangles AMD, ABC are ſimilar, ſince $\angle MDA = \angle BCA$, Cor. The. 20. Hence AC : BC : : AD : DM, by Th. 11. and AC × DM = BC × AD.

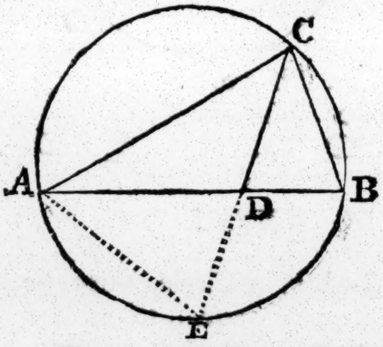
Again, the Triangles BAM, ACD are ſimilar,

For 1 $\angle ABM = \angle ACD$.
And 2 $\angle BAM = \angle CAD$.
Becauſe 3 $\angle MAD = \angle BAC$, and $\angle CAM$ common to both Triangles.
Theref. 4 $AC : CD :: AB : BM$, by Th. 11.
Hence 5 $AC \times BM = CD \times AB$, to which add the equal Quantities before found, and we have
6 $CD + AB + BC \times AD = AC \times BM + AC \times DM = AC \times \overline{BM + DM} = AC \times BD$. *Q.E.D.*

THEOREM XXIX.

The Square of the right Line biſecting any Angle of a Triangle and terminating in the oppoſite Side, together with the Rectangle under the two Segments of that Side, is equal to the Rectangle of the Sides including the propoſed Angle.

DEMONSTRATION.



Circumſcribe the Triangle ABC with a Circle, produce CD to meet the Circumference in E, and join AE, then are the Triangles ACE, BDE ſimilar,

For 1 $\angle E = \angle B$, by Cor. Th. 20.
And 2 $\angle ACE = \angle DCB$, by Hypotheſis.
Hence 3 $CE : CB :: AC : CD$, by Th. 11.
Theref. 4 $CE \times CD = CB \times AC$.
But 5 $CD \times CB = CD \times \overline{CD + DE}$.
$= \overline{CD}^2 + \overline{DE \times CD} = \overline{CD}^2 + AD \times DB$, by Th. 23
Conſeq. 6 $AC \times CB = \overline{CD}^2 + AD \times DB$. *Q.E.D.*

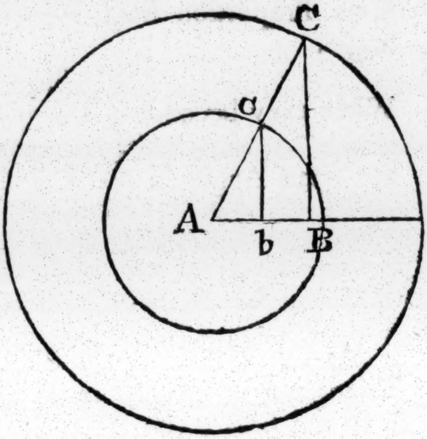
THEOREM XXX.

The Hypotenuſe of any two ſimilar right angled Triangles being made the Radius of two Circles, the Perpendiculars and Baſes in each will be in proportion to each other, as the Sines of their oppoſite Angles, anſwering each Radius.

DEMONSTRATION.

Let the two ſimilar right angled Triangles be ABC, and Abc, making the Hypotenuſe the Radius, then BC is the Sine $\angle A$, when AC is made Radius; and bc is the Sine of $\angle A$ when Ac is made the Radius; then by Theo. 11. $Ab : AB :: bc : BC$.

Q. E. D.



THEOREM XXXI.

If the Baſes of any two ſimilar right angled Triangles be made the Radii of two Circles, the Perpendiculars and the Hypotenuſes in each will be in proportion to the Baſes as the Tangents and Secants reſpectively of the Angle at the Center, anſwering each correſponding Radius.

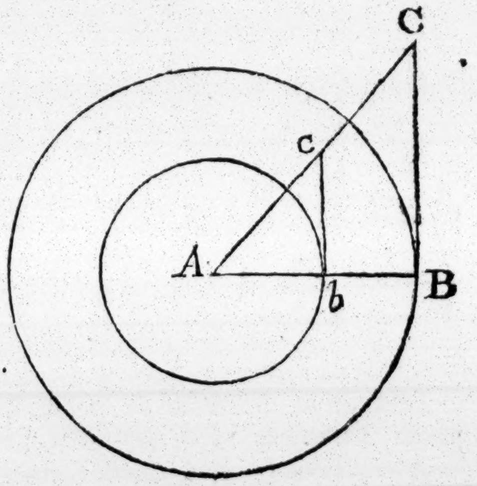
DEMONSTRATION.

Let the two ſimilar right angled Triangles ABC, Abc be propoſed, then by The. 11.

$Ab : bc :: AB : BC$.

$Ab : Ac :: AB : AC$.

Q. E. D.



Corollary. Whatever proportion any Side of a given right angled Triangle that is made the Radius or Semidiameter of a Circle, hath to the aſſumed Radius 10000000000 in the Tables of Sines, Tangents and Secants, the very ſame Proportion hath the other Sides of the given Triangle.

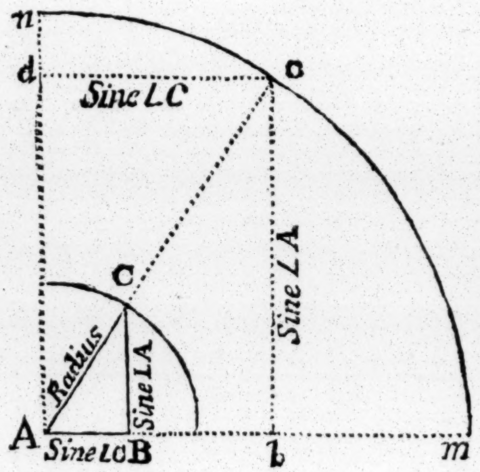
When they become $\left\{\begin{array}{c}\text{Sines}\\ \text{Tangents}\\ \text{Secants}\end{array}\right\}$ to the $\left\{\begin{array}{c}\text{Sines}\\ \text{Tangents}\\ \text{Secants}\end{array}\right\}$ of the ſame Angle in the Tables.

THEOREM XXXII.

Or *Axiom* 1. of Plane Trigonometry.

As the Word on the Side given in any right angled Triangle from the Table of Sines, Tangents and Secants, is to the Side given, ſo is the Word on the Side required from the ſame Tables, to the Side ſought.

DEMONSTRATION.



I. When the Hypotenuſe is made Radius. Let *mn* be a Quadrant of the tabular Circle whoſe Radius A*c* is 10000000000, and from thence the whole Canon of Sines, Tangents and Secants are conſtructed and diſpoſed into Tables.

$\left.\begin{array}{l} bc=\text{Sine}\angle A\\ Ab=\text{Sine}\angle C\end{array}\right\}$ from the Tables $\left\{\begin{array}{l}AC=\text{Radius}\\ BC=\text{Sine}\angle A\\ AB=\text{Sine}\angle C\end{array}\right\}$ in the Triangle ABC.

And ſince the tabular Triangle A*bc* is ſimilar to the given Triangle ABC, we have

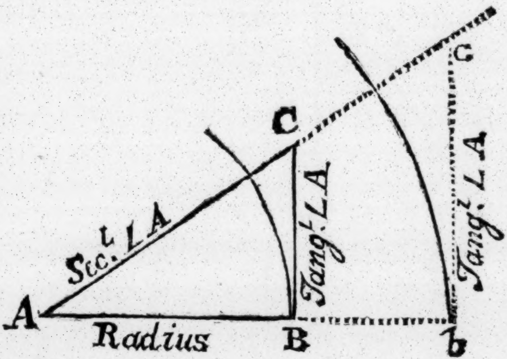
$Ac : AC :: bc : BC.$ *i. e.* $\text{Radius} : AC :: \text{Sine}\angle A : BC.$

$Ab : AB :: bc : BC.$ *i.e.* $\text{Sine}\angle C : AB :: \text{Sine}\angle A : BC.$

II. Making

II. Making either Leg Radius.

Here A*bc* is the tabular Triangle, and ABC is the Triangle given to be ſolved.



A*b* the Radius
bc Tangent ∠A } from the Tables.
A*c* Secant ∠A

AB the Radius
BC Tangent ∠A } in the Triangle given.
AC Secant ∠A

And ſince the Triangles ABC, A*bc*, are ſimilar, it will be

A*c* : AC : : A*b* : AB. *i. e.* Secant ∠A : AC : : Radius : AB.

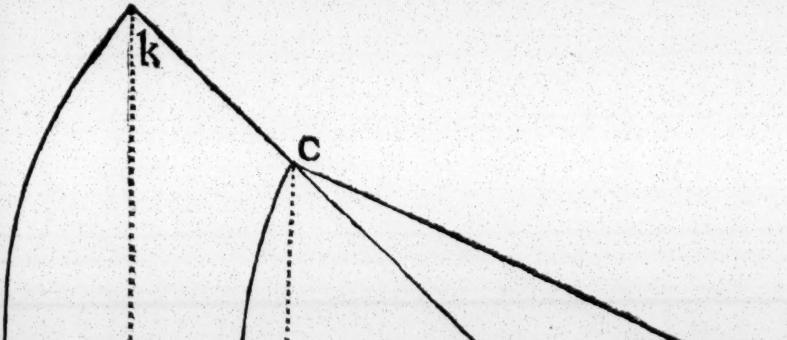
A*b* : AB : : *bc* : BC. *i. e.* Radius : AB : : Tangent ∠A : BC.
Q. E. D.

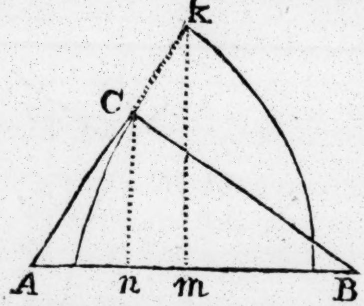
THEOREM XXXIII.

Or *Axiom* 2. of Plane Trigonometry.

The Sides of any plane Triangle, are in proportion to one another as the Sines of their oppoſite Angles.

DEMONSTRATION.





Produce the Side A C to k, ſo that $Ak = BC$, and let fall the Perpendiculars Cn, km, then will $km =$ Sine $\angle A$, and $Cn =$ Sine $\angle B$. Now the Triangle AnC is ſimilar to the Triangle Akm.

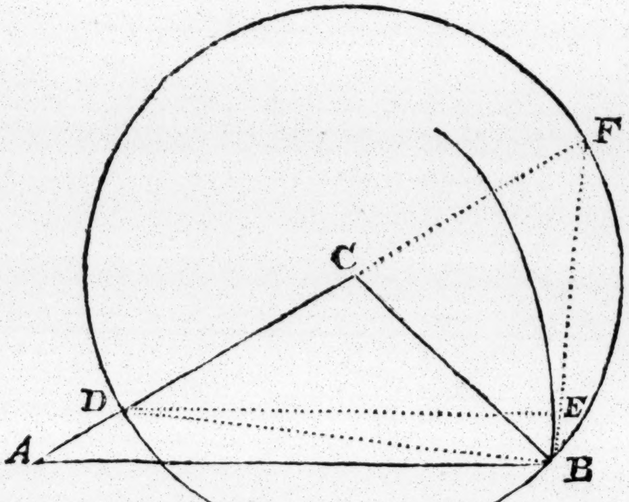
Therefore $AC : Cn :: Ak = BC : mk$, that is,
As AC : is to the Sine of its oppoſite Angle B : :
So is BC : to Sine of its oppoſite $\angle A$. *Q. E. D.*

THEOREM XXXIV.

Or *Axiom* 3. in Plane Trigonometry.

In any plane Triangle, as the Sum of any two Sides is to their Difference, ſo is the Tangent of half the Sum of their oppoſite Angles, to the Tangent of half their Difference.

DEMONSTRATION.



Let A B C repreſent the Triangle, on the Center C with the Radius C B deſcribe a Circle; produce A C to F, join F B, B D, and draw D E parallel to A B, then the Sum of the Sides AC, BC=AF, and Difference of the Sides AC, BC=AD.

But 1 $\angle A + \angle ABD = \angle BDC = \angle DBC$.

$1 + \angle ABD$ 2 $\angle A + \angle ABD + \angle ABD = \angle DBC + \angle ABD$.

That is, 3 $\angle A + 2\angle ABD = \angle ABC$.

Hence 4 $2\angle ABD =$ diff. $\angle\angle$'s A and ABC. half their Difference

Is 5 $\angle ABD = \angle BDE$, whoſe Tangent is BE.

Again 6 $\angle A + \angle ABD = \angle BDC$,

And 7 $\angle BDC = \angle DBC$, it will

Be 8 $\angle A + \angle ABD + \angle DBC = \angle BDC + \angle DBC$,

i. e. 9 $\angle A + \angle ABC = 2\angle BDC$.

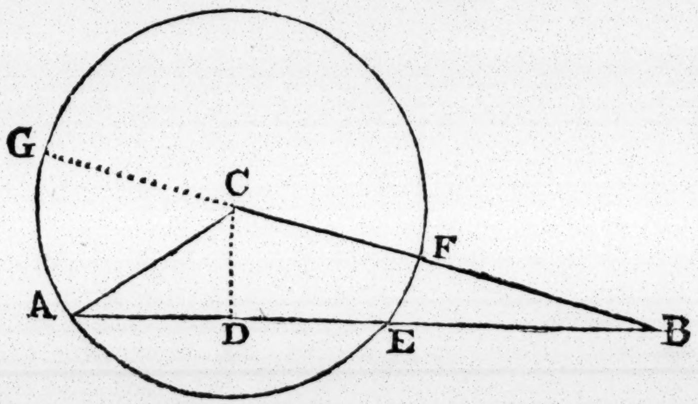
Hence 10 $\angle BDC =$ half the Sum $\angle\angle$'s A, and ABC, the Tanget of which is BF; then ſince the Triangles DBE, ABF are ſimilar, it will be

FA : DA : : FB : EB. *Q. E. D.*

THEOREM XXXV.

Being *Axiom* 4. Plane Trigonometry.

In every plane Triangle, as the Baſe is to the Sum of the other two Sides, ſo is the Difference of thoſe Sides to the Difference of the Segments of the Baſe, made by a Perpendicular let fall from the Angle oppoſite to the Baſe.



DEMONSTRATION.

Upon C as a Center with the Radius A C deſcribe a Circle, then the Sum of the Sides AC, CB is $=$GB, and their Difference, that is C B$-$A C$=$F B; alſo it is evident that E B is the Difference of D B and A D the Segments of the Baſe; then what is demonſtrated in Theorem XXIV. we have

$$AB : BF :: BG : EB.$$
$$AB : BG :: BF : EB.$$

Q. E. D.

FH
MVSEVM
BRITANNICVM

OF

LOGARITHMICAL ARITHMETIC.

Logarithmical Arithmetic ſhews how theſe Parts of Vulgar and Decimal Arithmetic that conſiſt of *Multiplication* and *Diviſion*, may be performed by *Addition* and *Subtraction* only.

Logarithms are artificial Numbers which are diſpoſed into Tables and placed againſt their natural Numbers, proceeding from 1 to 10000, and are ſuch that the *Sum* of any two Logarithms correſponds to the *Multiplication* of their natural Numbers, that is, the *Sum* of any two Logarithms ſtands againſt that Number which is the *Product* of thoſe Numbers anſwering to the two given Logarithms.

Thus,

Let the Number 84 its Logarithm 1.92427
be multiplied by 5 — — — 0.69897

The Product is 420 ſtand^{s} agſt its Log. 2.62324 the Sum.

The whole Number (as 2 in this Example prefixed to the decimal Part of any Logarithm, is called the *Index* of that Logarithm, and is always leſs by Unity than the Places of whole Numbers, in the Number anſwering to that Logarithm.

Thus 5742 the Logarithm is 3.75906
574.2 — — — 2.75906
57.42 — — — 1.75906
5.742 — — — 0.75906
.5742 — — — $\overline{1}$.75906

PROBLEM I.

To find the Logarithm of any whole Number.

RULE.

Find the Number in the left hand Column, in the Tables, and right againſt it, is its Logarithm required.

Examples.

4273, its Logarithm by the Rule is 3.6307329
765 — — — — — — 2.8859263
41 — — — — — — 1.6127839
8 — — — — — — 0.9030900

PROBLEM II.

To find the Logarithm of any mixed Number.

RULE.

Take the decimal Part of the Logarithm anſwering the given mixed Number, as if a whole Number; to which prefix an Index which muſt always be leſs by Unity than the Places of whole Numbers in the given mixed Number.

Examples.

371.7, its Logarithm is 2.5701926
41.17 — — — — 1.6145809
3.72 — — — — 0.5705429

PROBLEM III.

To find the Logarithm of any whole or mixed Number that exceeds the Limits of the Tables.

RULE.

Take the Difference between the two Logarithms in the Tables, which are the next greater and next leſs than your given Number; multiply this Difference by the Figures in the given Number, which are more than the Tabular Figures, cut off as many Figures from the right Hand of the Product as you multiply by, the reſt being added to the leſſer Logarithm, is the Anſwer.

Example I.

Required the Logarithm of 471285.

Operation.

471200 Logarithm — 5.6732053
471300 — — — 5.6732974

921
.85

4605
7368

782.85
5.6732053

471285 Logarithm = 5.6732835 *Anſwer.*

Example II.

Required the Logarithm of 825.814.

825.8 Logarithm — 2.9168749
825.9 — — — 2.9169275

526
.14

2104
526

73.64
2.9168749

825.814 Logarithm = 2.9168822 *Anſwer.*

PROBLEM IV.

Any Logarithm being given to find the Number anſwering thereto.

RULE.

Find the given Logarithm in the Tables, and right againſt it is the Number required

Example.

Required the Number anſwering the Log. 2.6732053 ?

Examine the Tables, and you will find the Number to be 471.2.

But when the Number anſwering the given Logarithm exceeds the Limits of the Tables, then find the Difference between the given Logarithm and the neareſt leſs in the Tables, to which add Cyphers, and divide the Reſult by the Difference between the next greater and next leſſer tabular Logarithm, the Quotient (which will conſiſt of as many Figures as you brought down Cyphers) being annexed to the right hand of the Number anſwering to the next leſs Logarithm, is the Anſwer.

Example I.

Required the Number anſwering the Log. 3.7284153?

Operation.

Next greater Logarithm — 3.7284349
Next leſs Logarithm — — 3.7283538

Difference — — — — 811

Given Logarithm — 3.7284153
5350.00 Logarithm — 3.7283538
75
811).615(.75
5350.75 *Anſwer.*

Example II.

Required the Number anſwering to the Log. 6.4213713?

Operation.

Next greater Logarithm — 6.4214394
Next leſs Logarithm — — 6.4212748

Difference — — — — 1646

Given Logarithm — 6.4213713
2638000.0 — — — 6.4212748
586.2
1646).965.0000(586.2
2638586.2 *Anſwer.*
8230
14200
13168
&c.

MULTIPLICATION *by Logarithms.*

CASE I.

To find the Product of two given whole or mixed Numbers.

RULE.

Find the Logarithm of each given Number, and their Sum will be the Logarithm of the Product, whoſe correſponding Number had from the Tables, is the Anſwer.

Example I.

Multiply	84	Logarithm	1.92427
by	25	— —	1.39794
Product	2100	— —	3 32221

Example II.

Multiply	41.5	Logarithm	1.61804
by	7.24	— —	0.85973
Product	300.4		2.47777

CASE II.

When both or either of the Factors are leſs than Unity.

RULE.

Remove the decimal Point towards the right Hand till the firſt Figure becomes a whole Number, with which take out the Logarithms, and find the Product as by *Caſe* I. then remove the decimal Point as many Places towards the left Hand as you before removed it to the Right, and it is the true Product.

Example I.

			removed.		
Multiply	5.4	—	5.4	Logarithm	0.73239
by	— .25	—	2.5	— —	0.39794
Product	1.35		13.5	— —	1.13033

Example II.

			removed.		
Multiply	.74	—	7.4	Logarithm	0.86923
by	5.25	—	5.25	— —	0.72016
Product	3.885		38.85		1.58939

Example III.

			removed.		
Multiply	.742	—	7.42	Logarithm	0.87040
by	.421	—	4.21	— —	0.62428
Product	.312382		31.2382		1.49468

DIVISION *by Logarithms.*

CASE I.

To divide a whole or mixed Number by a leſſer whole or mixed Number.

RULE.

From the Logarithm of the Dividend ſubtract the Logarithm of the Diviſor, and the Remainder is the Logarithm of the Quotient.

Example I.

Divide	—	624	Logarithm	2.79518
by	— —	26	— —	1.41497
Quotient		24	— —	1.38021

Example II.

Divide	—	1221	Logarithm	=3.08671
by	— --	81.4	—	=1.91062
Quotient	=	15	—	=1.17609

Example III.

Divide	—	34.86	Logarithm	=1.54232
by	— —	8.3	—	=0.91907
Quotient	=	4.2	—	=0.62325

CASE

CASE II.

When both or either of the Factors are leſs than Unity.

RULE.

Remove the decimal Points till the Factors contain whole Numbers, and the Dividend the greateſt, with which find the Product as before.

Then if the Dividend be more Places removed than the Diviſor, by the Exceſs remove the Quotient to the left Hand. But if the Diviſor be more Places removed, by the Exceſs remove the Quotient to the Right, and it is done.

Example I.

			removed		
Divide	72.42	—	72.42	Logarithm	=1.85985
by —	·543	—	5·43	— —	=0.73480
Quotient	1.81069		1.81069	— —	=1.12505

Example II.

			removed.		
Divide	8.75	—	87.5	Logarithm	=1.94200
by —	51.43	—	51.43	— —	=1.71121
Quotient	.17013		1.7013		0.23079

Example III.

			removed.		
Divide	.7824	—	7.824	Logarithm	=0.89342
by —	.4321	—	4.321	— —	=0.63558
Quot.	1.81069	—	1.8106	— —	=0.25784

PROBLEM V.

To Square, Cube, &c. any given Number by Logarithms.

RULE.

Multiply the Logarithm of the given Number, by the Index of the Power ſought.

Note, 2 is the Index of the Square, or ſecond Power.
3 is the Index of the Cube, or third Power.
&c. *&c.* *&c.*

Example I.
What is the Square of 51?
51 Logarithm = 1.70757
Index — — 2

Anſwer 2601 = 3.41514

Example II.
What is the Cube of 12?
12 Logarithm = 1.07918
Index — — 3

Cube is 1728 = 3.23754

PROBLEM VI.

To extract the Roots in Logarithms.

RULE.

This is only the Reverſe of the former Rule, namely, divide the Logarithm of the given Number by the propoſed Index, the Number anſwering to the Quotient is the required Root.

Example I.
What is the Square Root of 144?
144 Logarithm $=\overset{2}{|}2.15836$

Anſwer 12 = 1.07918

Example II.
What is the Cube Root of 1728?
1728 Logarithm $=\overset{3}{|}3.23754$

Anſwer 12 = 1.07918

To work the Rule of Three Logarithmically.

Add the Logarithms of the ſecond and third Terms together, from which Sum take the Logarithm of the firſt Term, and the Remainder is the Logarithm of the fourth Term ſought, where Radius is 10. As is evident to any one that peruſes the Solutions to the following Caſes in Trigonometry.

PLANE

PLANE TRIGONOMETRY.

TRigonometry is a Rule by which we learn how to compute the Length of the Sides and Quantity of the Angles of all manner of plane Triangles, from proper *Data*.

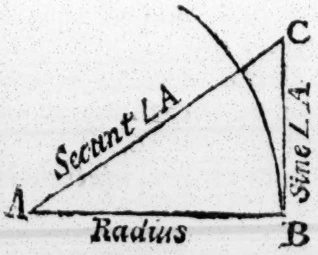
In order to which, there is calculated and diſpoſed into Tables, the Length of the Sines, Tangents and Secants of each Degree and Minute of the Quadrant of a Circle whoſe Radius is 10000000000, and theſe are called *natural* Sines, Tangents and Secants.

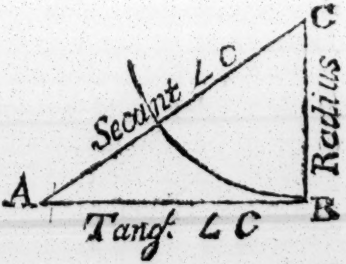
But with regard for accommodating the aforeſaid Tables to Practice, the Logarithms of the natural Sines, Tangents and Secants are diſpoſed into Tables, which are called *artificial* Sines, Tangents and Secants. The Logarithms I made uſe of in theſe Sheets are taken from Mr *Emerſon*'s Tables in his Book of Navigation, or *Art of Sailing on the Sea*, the decimal Part of which are carried to five Places only, being abundantly ſufficient for moſt practical Uſes.

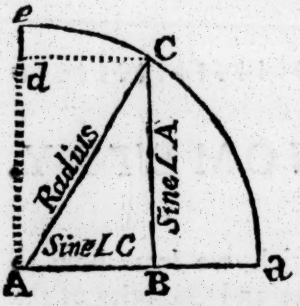
AXIOMS.

I. In right angled Triangles, any Side being made the Radius, Semidiameter or Sweep of a Circle, the other two will be Sines, Tangents or Secants, which Words being writ upon, or expounded by them, it will be

As the Word on the given Side,
Is to the given Side;
So is the Word on the Side required,
To the Side required.







N. B. The Word on any Side of a Triangle, is to be underſtood the Sine, Tangent or Secant of the Degrees and Minutes which are contained in the Angle expreſſed by that Word.

II. The Sides of any plane Triangle, whether right or oblique angled, are in proportion to one another, as the Sines of their oppoſite Angles.

III. In any plane Triangle

As the Sum of any two Sides
Is to their Difference,
So is the Tangent of half the Sum of their oppoſite Angles
To the Tangent of half their Difference.

IV. In any plane Triangle

As the Baſe is to the Sum of the two other Sides, ſo is the Difference of thoſe Sides to the Difference of the Segments of the Baſe, made by letting fall a Perpendicular from the Angle oppoſite the Baſe to the Baſe.

V. Half the Sum of any two Numbers being added to half their Difference, gives the greater Number. But,

Half the Difference taken from the half Sum, gives the leſſer Number.

VI. Having the three Sides of an oblique Triangle given, to find an Angle.

Set down the Side oppoſite to the Angle required, and under it the other two Sides; from their half Sum ſubtract each Side in the order they ſtand, then find the complement Arithmetical of the Logarithms of the half Sum, and firſt Remainder, and alſo the Logarithms of the other two Remainders, then will the half Sum of theſe four Logarithms be the Tangent of half the Angle ſought.

The *Arith. Com.* of a Logarithm is found by beginning at the Index, and writing down what each Figure wants of 9, all but the laſt, which muſt be from 10.

When there are two Figures in the Index, reject the firſt, and work as before.

VII. Any two Sides of a plane Triangle taken together, are greater than the third that remains.

VIII. The greateſt Side of every Triangle is oppoſite to the greateſt Angle, and the greateſt Angle oppoſite the greateſt Side.

IX. The Sum of the three Angles of every plane Triangle is equal to 180°, therefore

X. If two Angles of a plane Triangle are known, the third is alſo known, being found by taking their Sum from 180 Degrees.

XI. If any Angle be obtuſe or greater than 90 Degrees, each of the other two will be acute or leſs than 90 Degrees.

XII. If one Angle be right or 90 Degrees, the Sum of the other two will be 90 Degrees.

XIII. If one of the acute Angles, in a right angled plane Triangle, be given, the other is alſo known, being ſound by taking the given Angle from 90°, and this Defect is called its Complement.

CASE I.

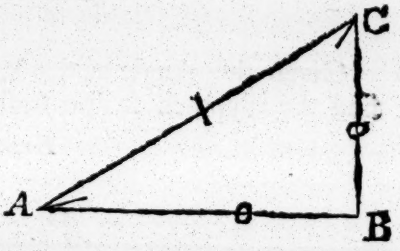
The Hypotenuſe and Angle at the Baſe being given, to find the Baſe and Perpendicular.

It will be proper to obſerve,

1. When a Side is required, begin with an Angle oppoſite to a given Side.
2. When an Angle is required, begin the Proportion with a Side oppoſite to a given Angle.

Example.

In the Triangle ABC, right angled at B.



Are given $\left\{ \begin{array}{l} AC=34 \\ \angle A=28^\circ : 4' \end{array} \right\}$

Required AB and BC.

Geometrical Solution.

Draw any right Line AB, make ∠A = 28° 4′, and draw the Hypotenuſe AC = 34 from any Scale of equal Parts, fitted to your Line of Chords, from the Point C, let fall the Perpendicular BC, and it is done.

Meaſure $\left\{ \begin{matrix} AB = 30 \\ BC = 16 \end{matrix} \right\}$ on the ſame Scale of equal Parts.

Numerical Solution.

From 90°
take ∠A = 28 : 4′

∠C = 61 : 56 by Ax. 13.

To find BC.

As Radius, Sine 90° (∠B) - - - - -	10.00000
To Hypotenuſe AC = 34, Log. - - -	1.53148
So is Sine ∠A 28° : 4′ - - - - -	9.67255
To BC = 16 Log. - - - - - -	1.20403

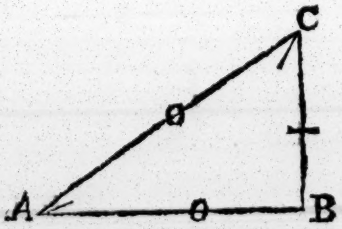
To find AB.

As Radius, Sine 90° (= ∠B) - -	10.00000
Is to Hypot. AC = 34 Log. - - -	1.53148
So is Sine ∠C = 61° 56′ - - - -	9.94566
To AB = 30 Log. - - - - - -	1.47714

CASE II.

Given the Angle at the Baſe and the Perpendicular, to find the Hypotenuſe and Baſe.

Example.



In the Triangle ABC, right angled at B.

Are given $\left\{\begin{array}{l} BC=12 \\ \angle A=36^\circ:52' \end{array}\right\}$ Required AC, AB.

Geometrical Solution.

At any Point B draw BC perpendicular to AB, which make =12, from a Scale of equal Parts, then from the Line of Chords make the ∠C=53° 8′ (for 90°—36° 52′ =53° 8′) draw the Hypotenuſe CA, and where it meets the Line AB the Triangle is conſtructed, which meaſured from the ſame Scale of equal Parts the Perpendicular was laid down, you will find AC=20, and AB=16.

Numerical Solution.

1*ſt*, To find AC.

From 90°
take ∠A= 36 : 52′
Remains ∠C=53 : 08

As Sine ∠A=36° 52′	9.77812
Is to Perpendicular BC=12	1.07918
So is Radius 90°	10.00000
To Hypotenuſe AC=20	1.30106

To find AB.

As Sine ∠A=36° 52′ — Com. Arith.	0.22188
Is to BC=12 — Log.	1.07918
So is Sine ∠C=53° 8′	9.90310
To AB=16	1|1.20416

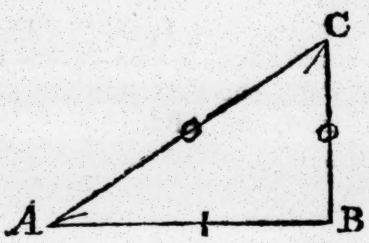
Or AB may be found thus, now AC is found.

As Radius 90°	10.00000
Is to the Hypotenuſe AC=20	1.30106
So is Sine ∠C=53° 8′	9.90310
To AB=16	1.20416

CASE III.

The Angles and Baſe given, to find the Hypotenuſe or Perpendicular.

Example.



Let the Baſe $AB = 36$, and $\angle A = 36^\circ\ 52'$.

Required the Hypotenuſe AC and Perpendicular BC.

Geometrically.

Draw the Baſe AB which make $= 36$, from the equal Parts, raiſe the Perpendicular BC, make $\angle A = 36^\circ\ 52'$ from the Chords, then draw AC, and where it cuts the Perpendicular BC in C, the Thing is done; and AC meaſures 45, $BC = 27$.

Calculation.

From 90°
take $\angle A = 36 : 52'$

Remains $\angle C = 53 : 08$

As Sine $\angle C = 53^\circ\ 8'$ — — — 9.90310

Is to Baſe $AB = 36$ Log. — — 1.55630
So is Radius Sine 90° — — — 10.00000

To Hypotenuſe $AC = 45$ — — 1.65320

As Sine $\angle C = 53^\circ\ 8'$ — — — — 9.90310

Is to Baſe $AB = 36$ — — — — 1.55630
So is Sine $\angle A = 36^\circ\ 52'$ — — — 9.77812

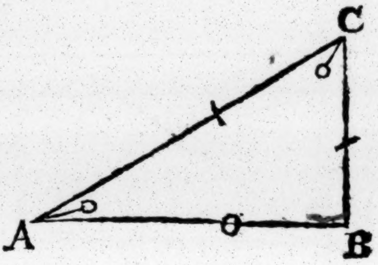
To the Perpendicular $BC = 27$ — 1.43132

CASE IV.

Given the Hypotenuſe and Perpendicular, to find the Baſe and Angles.

Example.

In the Triangle ABC, right angled at B.



Are given { AC=40 / BC=24 } Required AB and ∠∠'s A & C.

Geometrically.

Draw AB, upon which raiſe the Perpendicular BC=24, then with the Extent of 40=AC with one Foot in C cut BA in A, join AC, and it is done, and AB meaſures 32 from the equal Parts and ∠A, 36° 52′ from the Line of Chords, wherefore 90°—36° 52′=53° : 8′=∠C.

Calculation.

As the Hypotenuſe AC=40 — — 1.60206

Is to Radius 90° — — — — 10.00000
So is the Perpendicular BC=24 — 1.38021

To Sine ∠A=36° 52′ — — — 9.77815

From 90°
take ∠A= 36 : 52′

Remains ∠C=53 : 8

As Radius 90° — — — — 10.00000

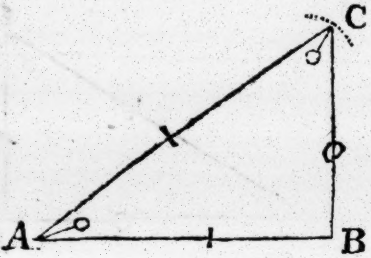
Is to AC=40 — — — — 1.60206
So is Sine ∠C=53° 8′ — — 9.90310

To the Baſe AB=32 — — — 1.50516

CASE V.

Given the Hypotenuſe and Baſe to find the Angles and Perpendicular.

Example.



Let Hypot AC=73 }
Baſe AB=55 }
Required ∠∠'s A, C,
and Perpendicular.

Geometrical Solution.

From the equal Parts draw the Baſe AB=55, at B erect the Perpendicular BC, then with the Extent of 73, and one Foot in A, cut BC in C, draw AC, and it is done.

Meaſured { BC=48° }
{ ∠A=41 : 7′ } Then { 90° From
{ 41 : 7′ take ∠A.

Remains 48 : 53=∠C.

Calculation.

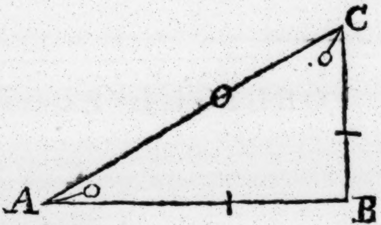
As the Hypotenuſe AC=73 — —	1.86332
Is to Radius 90° — — —	10.00000
So is Baſe AB=55 — — —	1.74036
To Sine ∠C=48° 53′ — —	9.87704
As Radius 90° — — — —	10.00000
Is to AC=73 — — — —	1.86332
So is the Sine of ∠A=41° 7′ —	9.81795
To the Perpendicular BC=48 —	1.68127

CASE VI.

Given the Baſe and Perpendicular, to find the Hypotenuſe and Angles.

Example.

Let Baſe AB=45 }
Perpendicular BC=28 }
Required AC and ∠∠'s A and C.



Geometrical Solution.

From the equal Parts make AB=45, raiſe the Perpendicular BC which make =28 from the ſame Scale, join the Points A and C, and it is done.

Meaſured { AC=53
∠A = 31° : 53′ take
90 : 00 from

∠C=58 : 7 Remains.

Calculation.

As Baſe AB=45 —— —— 1.65321

Is to Radius Tangent 45° —— 10.00000
So is the Perpendicular BC=28 — 1.44715

To the Tangent ∠A=31° 53′ — 9.79394

From 90°
take ∠A= 31 : 53′

Remains ∠C=58 : 7

As Sine ∠C=58° 7′ —— —— 9.92897

Is to the Baſe AB=45 —— 1.65321
So is Radius Sine 90° —— 10.00000

To the Hypotenuſe AC=53 —— 1.72424

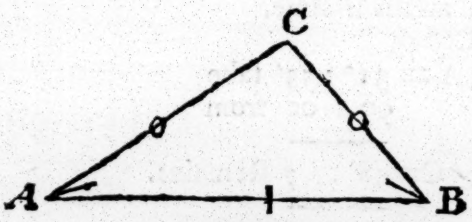
Obſervation. By making the Hypotenuſe Radius, each Side becomes Sines of their oppoſite Angles, and therefore the Sides are in proportion to the Sines of their oppoſite Angles, and the contrary by Axiom 2, and this muſt be allowed the moſt elegant when it can be uſed; which I have done in every Caſe except the laſt.

OBLIQUE PLANE TRIANGLES.

CASE I.

Given one Side and the Angles to find the other two Sides.

Example.



In the oblique Triangle ACB,

$$\text{Are given}\left\{\begin{array}{l}\angle A=28^\circ:4'\\ \angle B=53:8\\ AB=42\end{array}\right\}\text{Required AC and BC.}$$

Geometrical Conſtruction.

Make AB=42, draw AC making the Angle A=28° 4', alſo making the ∠B=53° 8', draw BC, and where it meets AC the Triangle is conſtructed.

Calculation.

$$\begin{array}{ll}\text{From} & 180^\circ:00'\\ \text{Take}\left\{\begin{array}{l}\angle A=28^\circ:4'\\ \angle B=\underline{53:8}\end{array}\right. & \\ \text{Sum}= & \underline{81:12}\\ \text{Remains } \angle C= & \underline{98:48}\end{array}$$

its Supplement is 81° 12' which uſe.

As Sine ∠C=98° 48′ — 9.99485

Is to AB=42 — — 1.62325
So is Sine ∠A=28° 4′ — 9.67255

To BC=20 — — 1.30095

As Sine ∠A=28° 4′ — — 9.67255

Is to BC=20 — — 1.30095
So is Sine ∠B=53° 8′ — 9.90310

To AC=34 — — 1.53150

Or you may find the Sides thus.

As Sine ∠C=98° 48′ — Co. Ar. 0.00514
Is to AB=42 — — 1.62325
So is the Sine ∠A=28° 4′ — 9.67255

To BC=20 — — 1|1.30094

As Sine ∠C=98° 48′ — Co. Ar. 0.00514
Is to AB=42 — — 1.62325
So is Sine ∠B=53° 8′ — 9 90310

To AC=34 — — 1.53149

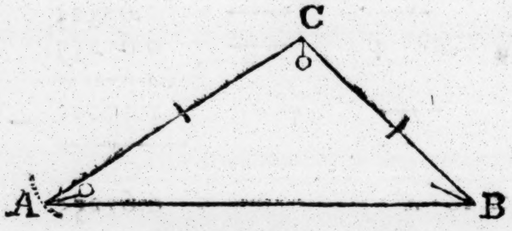
CASE II.

Two Sides and one Angle oppoſite to them being given, to find the reſt.

N. B. When the given Angle is oppoſite to the greater given Side, this Caſe has one certain Anſwer: But when the given Angle is oppoſite to the leſſer given Side, this Caſe will then have two true Anſwers, as will evidently appear by the following Examples.

Example

Example I.



In the oblique Triangle ACB,

Are given { AC=85°, BC=50, ∠B=53° : 8′ } Required AB and ∠∠'s C and A.

Geometrical Conſtruction.

Draw AB, and alſo BC=50 making ∠B=53° 8′, then with the Extent of 85, and one Foot in C, cut BA in A, and it is done.

Meaſured { AB=105, ∠A= 28° 4′ }

As AC=85 —— ——	1.92941
To Sine ∠B=53° 8′ ——	9.90310
So is BC=50 —— ——	1.69897
To Sine ∠A=28° 4′ ——	9.67266

From 180° : 00

Take { ∠A=28° : 4′
∠B=53 : 8 }

Sum= 81 : 12

Remains ∠C=98 : 48 by Ax. 10.

As Sine ∠B=53° 8′ ——	9.90310
Is to AC=85 —— ——	1.92941
So is Sine ∠C=98° 48′ ——	9.99485
To AB=105 —— ——	2.02116

Or thus.

As $AC=85$ — — — Co. Ar. 8.07058
To Sine $\angle B=53^\circ\ 8'$ — — — 9.90310
So is $BC=50$ — — — — 1.69897

To Sine $\angle A=28^\circ : 4'$ — — 9.67265
$\angle B=53 : 8$

Take 81 : 12
from 180 : 00

$\angle C=98 : 48$

As Sine $\angle B=53^\circ\ 8'$ — Co. Ar. 0.09689
Is to $AC=85$ — — — — 1.92942
So is Sine $\angle C=98^\circ\ 48'$ — — 9.99485

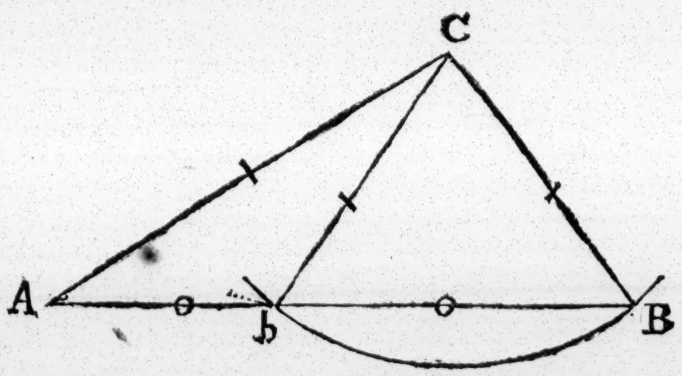
To $AB=105$ — — — — 2.02119

Example II.

$$\text{Let}\left\{\begin{array}{l} BC=50 \\ AC=85 \\ \angle A=28^\circ 4' \end{array}\right\}\text{Required } AB \text{ and } \angle\angle\text{'s } B \text{ and } C.$$

Geometrically.

Draw AB, and AC=85, making the $\angle A=28^\circ\ 4'$, then with the Extent of 50, and one Foot in C, cut AB, in the two Points B, *b*, join BC, *b*C, and either of the Triangles will be that required.



Meaſure $\begin{cases} AB=105 & Ab=45. \\ \angle ACB=98^\circ : 48' & \angle ACb=25^\circ : 4'. \end{cases}$

Calculation.

As $BC=50$ — — — Co. Ar. 8.30103
Is to the Sine $\angle A=28^\circ\ 4'$ — — 9.67255
So is $AC=85$ — — — — 1.92942

To Sine $\angle B=\angle CbB=53^\circ\ 8'$ — 9.90300

From — — — $180^\circ : 00'$
Take $\begin{cases} \angle B=53^\circ : 8' \\ \angle A=28 : 4 \end{cases}$

Sum 81 : 12

Remains $\angle ACB$ 98 : 48

From $\angle CbB=53^\circ : 8'$
take $\angle A=28 : 4$

Remains $\angle ACb=25 : 4$
And $\angle AbC=126 : 52$ by Ax. 10.

As Sine $\angle A=28^\circ\ 4'$ — Co. Ar. 0.32744
Is to $BC=50$ — — — — 1.69897
So is Sine $\angle ACB=98^\circ\ 48'$ — — 9.99485

To $AB=105$ — — — — 2.02126

As Sine $\angle A=28^\circ\ 4'$ — Co. Ar. 0.32744
Is to $BC=50$ — — — 1.69897
So is Sine $\angle ACb=25^\circ\ 4'$ — 9.62703

To $Ab=45$ — — — — 1.65344

CASE III.

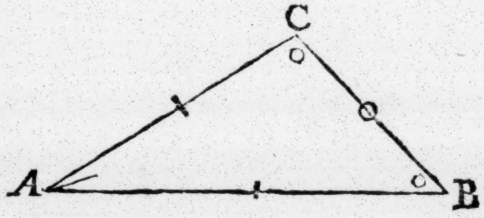
Two Sides and their contained Angle being given, to find the reſt.

Example.

Let $\left\{\begin{array}{l} AB=64 \\ AC=48 \\ \angle A=36^\circ\ 14' \end{array}\right\}$ Required BC and ∠∠C and B.

Geometrically.

Make AB = 64, and ∠A = 36° 14′, draw AC which make = 48, join BC, then will the Triangle ACB be that required, BC measures 38, and ∠B = 48° 18′.



Calculation by Axiom 3.

AB = 64
AC = 48

Sum Sides = 112

From 180° : 00′
take ∠A = 36 : 14

Sum = 143 : 46 = ∠∠'s B&C.

Half Sum = 71 : 53

As Sum Sides 112 — — Co. Ar. 7.95078
Is to their Diff. 16 — — — — 1.20412
So is Tang. half ∠∠'s B&C = 71° 53′ 10 48522

To Tangent half their diff. = 23° 35′ 9.64012

To half Sum = 71° : 53′
Add half Diff. = 23 : 35

Greater ∠C = 95 : 28

From half Sum = 71° : 53′
Take half Diff. = 23 : 35

Leſſer ∠B = 48 : 18

As Sine ∠B = 48° 18′ — Co. Ar. 0.12689
Is to AC = 48 — — — — 1.68124
So is Sine ∠A = 36° 14′ — — 9.77164

To BC = 38 — — — — -- 1.57977

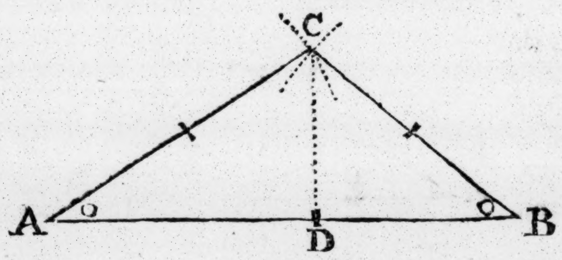
CASE IV.

The three Sides being given, to ſind the Angles.

Example.

Let $\left\{\begin{array}{l}AB=52\\ BC=32\\ AC=40\end{array}\right\}$ Required ∠'s A, C, and B.

Geometrically.



Make AB =52, with the Extent of 40, and one Foot in A, ſtrike an Arch, alſo with the Extent of 32, and one Foot in B, cut the former Arch in C, join AC and BC, and it is done. Make the Perpendicular CD.

Meaſure $\left\{\begin{array}{l}\angle A=37^\circ : 57'\\ \angle B=50 : 15\end{array}\right.$

Calculation by Axiom 4.

AC=40
BC=32

Sum=72

AC=40
BC=32

Diff.=8

Baſe. Sum. Diff.
As 52 : 72 :: 8 : 11.07=diff. Segments of the Baſe.

5.53=half diff. Seg.

To half Baſe =26
Add half diff. Seg.= 5.53

Sum=greateſt Seg. 31.53=AD.

From half Baſe =26
Take half diff. Seg. = 5.53

Remains leſſer Seg.=DB 20.47

Given $\left\{\begin{array}{l}AC \text{ the Hyp.}=40\\ AD \text{ the Baſe}=31.53\end{array}\right\}$ to find the ∠'s A & ACD.

As AC=40 — — — — — 1.60206

Is to Radius, Sine 90° — — 10.00000
So is AD=31.53 — — — — 1.49872

To Sine ∠ACD=52° : 01′ — — 9.89666
From 90 : 00

Remains ∠A=37 : 59
As BC=32 — — — — — 1.50515

Is to Radius, Sine 90° — — — 10.00000
So is BD=20.47 — — — — 1.31111

To Sine ∠BCD=39° : 46′ — — 9.80596
From 90 : 00

Remains ∠B=50 : 14
Now ∠C=∠ACD+∠BCD=91° 47′.
For ACD=52° : 01′
And BCD=39 : 46

91 : 47

Otherwiſe by Axiom 6.

BC=32
AB=52
AC=40

124

Half Sum=62 — — — — Co. Ar. 8.20760
Remainders { 30 — — — — Co. Ar. 8.52288
10 Log. — — — — 1.00000
22 — — — — — 1.34243

19.07291

Half is Tangent of 18° : 58½′ 9.53645
2

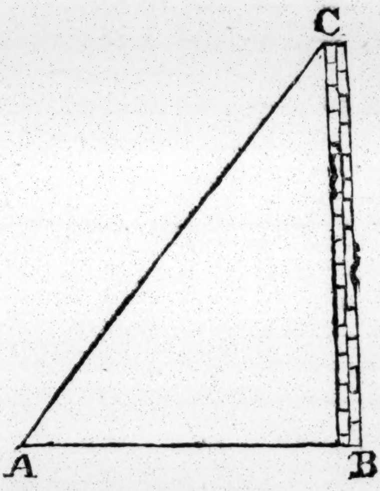
∠A=37 : 57 as above, hence the reſt will be found by Axiom 2.

I have purpoſely omitted the Solutions to the foregoing Caſes by *Gunter*'s Scale, it being extremely eaſy to any Perſon that can but handle a Pair of Compaſſes.

I ſhall now proceed to various Uſes of a right angled Triangle, and the Oblique, with regard to the taking of Heights and Diſtances, *&c.*

PROBLEM I.

To find the Height of a Tower, Caſtle, Church, &c. *when you can come to the Foot of it.*



With any Inſtrument to take Angles, as ſuppoſe a Quadrant, you ſtand ſome diſtance from the Foot of the Tower, as at A, and there taking your Quadrant and directing the Sights of it to C, and find the String cuts the Limb at 48° 12′, then find by meaſuring in a ſtraight Line how far you ſtood from the Foot of the Tower, and note the Meaſure, ſuppoſe here 45 Yards, then may the Height of the Tower be found.

BC denotes the Tower or Perpendicular of the right Angled Triangle ABC.

AB the Diſtance meaſured 45 Yards, or Baſe of the Triangle.

AC the viſual Sight from the Quadrant to the Tower's Top, or Hypotenuſe.

$$\text{Now there is given } \left.\begin{array}{l} \angle A = 48^\circ : 12' \\ \angle C = 41 : 48 \\ AB = 45 \text{ Yards} \end{array}\right\} \text{to find BC.}$$

By *Caſe* III.

As Sine $\angle C = 41^\circ\ 48'$ — — — 9.82382

Is to $AB = 45$ — — — — 1.65321
So is $\angle A = 48^\circ\ 12'$ — — — 9.87243

To $BC = 50.33$ — — — — 1.70182

The Height of the Inſtrument above Ground muſt always be added, which here ſuppoſe was 2 Yards above Ground, then

52.33 Yards is the Tower's Height.

PROBLEM II.

To find the Breadth of a River.

An Engineer comes to the Side of a River, as at B, where he is to lay a Bridge over it to paſs the Army, and wants to know the Breadth of it, in order to know how many Pontons he muſt uſe upon that Occaſion. Standing at B he ſees ſome Object, ſuppoſe at C, on the other Side of the River, and then making a right Angle with his Inſtrument, he obſerves another Object by the River Side at A, then meaſuring to it finds it 49 Yards, and the $\angle A =$ 53° 20′, hence the Breadth of the River may be found thus. *See the laſt Figure.*

$$\text{Given}\ \left.\begin{array}{l}\angle A = 53^\circ : 20' \\ \angle C = 36 : 40 \\ AB = 49\ \text{Yards}\end{array}\right\}\ \text{to find BC.}$$

As Radius — — — — — — 10.00000

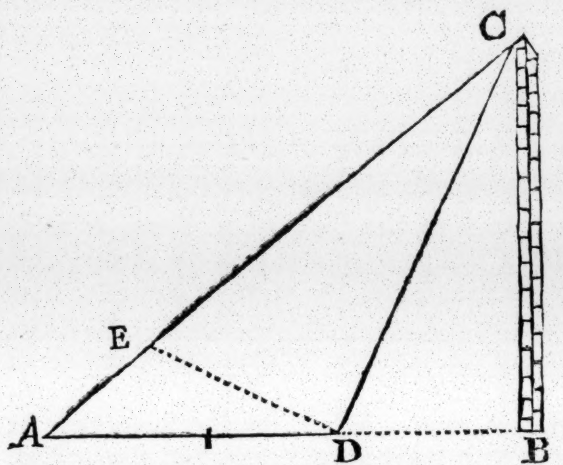
Is to AB, 49 — — — — — — 1.69019
So is Tangent $\angle A$, 53° 20′ — 10.12815

To BC=65.82 — — — — — 1.81834

But when the Caſe happens you cannot come to the Foot of a Tower, &c. by reaſon of Water, Enemies, or other Obſtacle, it is then called an inacceſſible Height, and may be found by the following

PROBLEM III.

To meaſure an inacceſſible Height.



At any convenient Diſtance from the Tower, as at A, find the Angle of Altitude, ſuppoſe 28° 34′, then meaſure in a ſtraight Line towards the Tower, as to D, and ſuppoſe it meaſures from A to D 30 Yards, then at D take another Angle of Altitude, which let be 50° 9′.

Here the ſight Lines AC, DC, and the diſtance Line AD form an oblique Triangle ADC, wherein are given the Diſtance AD=30, and all the Angles to find DC, and conſequently BC.

Firſt $AD=30$
$\angle CAB=28^\circ : 34'$
$\angle BDC=50^\circ : 9'$
$\angle DCB=39^\circ : 51'$ by Ax. 13.

But $\angle ADC=129^\circ\ 51'=\angle DCB+\angle B=39^\circ\ 51'+90^\circ$ by Theorem 8.

Then $\angle A = 28^\circ : 34'$
$\angle ADC=129 : 51$

Take $158 : 25$
from $180 : 00$

Remains $\angle ACD=21 : 35$ by Ax. 10.

As Sine ∠ACD=21° 35′ — — 9.56567

Is to AD=30 — — — — 1.47712
So is Sine ∠A=28° 34′ — — 9.67959

To ſight Line DC=39 — — 1.59104

As Radius — — — — — 10.00000

Is to Hyp. DC=39 — — — 1.59104
So is Sine ∠D=50° 9′ — — — 9.88520

To BC=30 ferè, Altitude — — 1.47624

N. B. You may make uſe of this Problem for finding the Length of unapproachable Lines, as thoſe of Places beſieged; but never make the Angle DCA leſs than two Degrees, and if Ground will permit, go from D to E at right Angles.

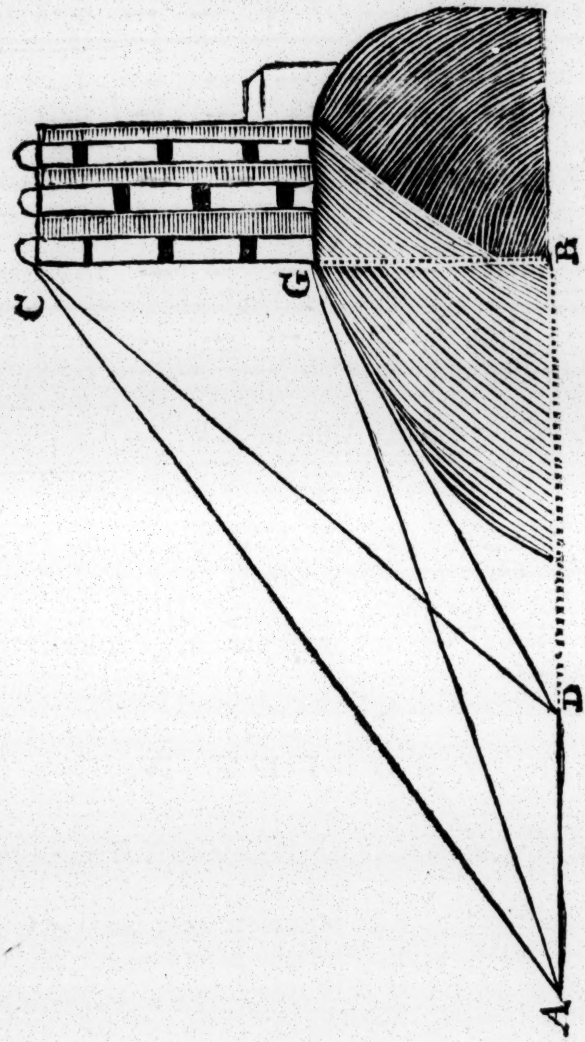
PROBLEM IV.

To find the Height of a Caſtle ſtanding upon a Hill.

Firſt find the ∠CAD ſuppoſe=40°, and ∠GAB=22°, then meaſure in a ſtraight Line towards the Caſtle, as from A to D, let be 40 Yards; then at D find ∠CDB, let=63° 20′, and ∠GDB=49°, and you may from theſe *data* find the Height.

⁂

Solution.



Solution.

$\angle CAD=40^\circ$
$\angle GAB=22^\circ$
$\angle CDB=63'\ 20'$
$\angle GDB=49^\circ$
$\angle DCB=26^\circ\ 40'$ by Ax. 13.
$\angle B\quad=90^\circ$
$\angle ADC=116^\circ\ 40'=\angle DCB+\angle B$, by *Theorem* 8.
$\angle ADC=23^\circ\ 20'$ by Ax. 10.
And $AD\quad=40$ Yards.

As Sine $\angle ACD=23^\circ\ 20'$ — — 9.59778

Is to $AD=40$ — — — — 1.60206
So is Sine $\angle CAD=40^\circ$ — — 9.80806

To ſight Line $CD=64.91$ — — 1.81234

As Radius — — — — — 10.00000

Is to $CD=64.91$ — — — — 8.81234
So is Sine $\angle CDB=63^\circ\ 20'$ — — 9.95115

To $BC=58$ the Hill and Caſtle — 1.76349

As Radius — — — — — 10.00000

Is to $CD=64.91$ — — — — 1.81234
So is Sine $\angle DCB=26^\circ\ 40'$ — — 9.65205

To $DB=29.13$ — — — — 1.46439

As Radius — — — — — 10.00000

To $DB=29.13$ — — — — 1.46439
So is the Tangent $\angle GDB=49^\circ$ — 10.06083

To $BG=33.51$ the Alt. Hill — 1.52522

From Altitude of Hill and Caſtle $BC=58$ Yards.
Take BG the Perpend. Hill — — 33.51

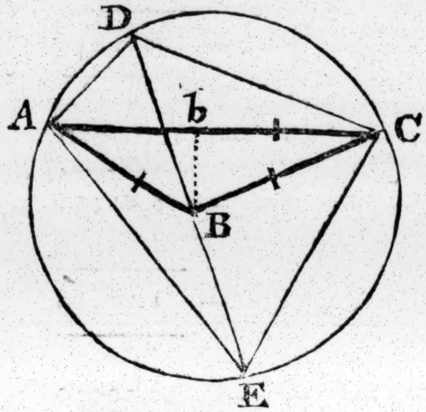
Remains CG the Caſtle's Height$=24.49$

PROBLEM V.

Caſe I.

Let A, B, C be three Churches whoſe Diſtances are, *viz.* $AC=106$, $BC=65.5$, $AB=53.25$, and ſuppoſe at ſome

Station, as at E, I can ſee all thoſe Objects, and would know their reſpective Diſtances from me, I take an Angle AEB$=13^\circ\ 30'$, alſo $\angle$CEB$=29^\circ\ 50'$, from theſe *data* their Diſtance from me may be found as follows,



Through A, C and E deſcribe a Circle, and draw AE, CE, and BE, continue BE to D, and join AD, DC, then there are given

$AC=106$
$BC=65.5$
$AB=53.25$
$\angle AEB=13\ 30'=\angle AED.$
$\angle CEB=29^\circ\ 50'=CED$
$\angle AEC=43^\circ\ 20'=\angle AEB+\angle CEB.$
$\angle ADC=136^\circ\ 40'=180-\angle AEC$, by Theo. 8.
$\angle ACD=13^\circ\ 30'=\angle AED$ } by *Cor.* Th. 20.
$\angle CAD=29^\circ\ 50'=\angle CED$ }

Make the Perpendicular B*b*.

1. In the Triangle ABC, all the Sides are given to find the Angles. By Caſe IV.

$BC+AB=118.75$ the Sum of the Sides.
$BC-AB=12.25$ the Difference of the Sides.

As Baſe AC=106 — — — 2.02530

Is to the Sum Sides=118.75 — 2.07445
So is the Diff. 12.25 — — — 1.08813

To the diff. Seg. Baſe=13.71 — 1.13728

Now half diff. 6.855
Half Baſe 53

Greater Seg. *b*C= 59.855

Leſſer Seg. A*b*= 46.145

As $AB = 53.25$ — — — — 1.72632

Is to Radius 90° — — — — 10.00000
So is $Ab = 46.14$ — — — — 1.66407

To Sine $\angle ABb = 60^\circ\ 3'$ — — 9.93775
Then $\angle BAb = 29^\circ\ 57'$

As $BC = 65.5$ — — — — — 1.81624

Is to Radius — — — — — 10.00000
So is $bC = 59.85$ — — — — 1.77706

To Sine $\angle bBC = 66^\circ\ 2'$ — — 9.96082
Then $\angle BCb = 23^\circ\ 58'$

$$\begin{aligned} \text{And } \angle bBC + \angle ABb &= 126^\circ : 5' = \angle ABC. \\ &29 : 57 = \angle CAB. \\ &23 : 58 = \angle ACB. \end{aligned}$$

2. In the Triangle ADC.

$$\text{Are given} \left\{ \begin{array}{l} AC = 106 \\ \angle DAC = 29^\circ\ 50' \\ \angle ACB = 13^\circ\ 30' \\ \angle ADC = 136^\circ\ 40' \end{array} \right\} \text{to find } AD,\ DC.$$

As Sine $\angle D = 136^\circ\ 40'$ — — 9.83647

Is to $AC = 106$ — — — — 2.02530
So is Sine $\angle C = 13^\circ\ 30'$ — — 9.36818

To $AD = 36.06$ — — — — 1.55701

As Sine $\angle D = 136^\circ\ 40'$ — — 9.83647

Is to $AC = 106$ — — — — 2.02530
So is Sine $\angle A = 29^\circ\ 50'$ — — 9.69677

To $DC = 76.85$ — — — — 1.88560

And $\angle DCB = 37^\circ\ 28' = \angle ACD + \angle ACB$.

3. In the Triangle BCD, are given two Sides and the contained Angle to find the Angles, by *Caſe* III.

AC=76.85
BC=65.5 } to find the Angles D and B.
∠DCB=37° 28′

DC+BC=142.35 Sum Sides.
DC—BC=11.35 Diff. Sides.
180°—∠37° 28′=142 : 32′ Sum Angles D and B.

71 : 16=half Sum.

As Sum Sides 142.35 — — — 2.15320

Is to the Diff. 11.35 — — — 1.05499
So is Tang. half Sum=71° 16′ — 10.46963

To Tang. half Diff.=13° : 14′ — 9.37142
Half Sum=71 : 16

∠BDC=84 : 30=greater Angle.

∠DBC=52 : 02=leſſer Angle.

From 180°
84 : 30

∠CBE= 95 : 30
∠CEB= 29 : 50

125 : 20

From 180°
125 : 20

∠ECB=54 : 40

4. In the Triangle EBC.

Are given { BC=65.5
∠CBE=95° : 30′
∠CEB=29 : 50
∠ECB=54 : 40 by Ax. 10. } to find BE, EC.

As Sine ∠E=29° 50′ — — — 9.69677

Is to BC=65.5 — — — — 1.81624
So is Sine ∠C=54° 40′ — — 9.91158

To BE=107.4 — — — — 2.03105

As Sine $\angle E = 29^\circ\ 50'$ — — — 9.69677

Is to $BC = 65.5$ — — — — 1.81624
So is Sine $\angle B = 95^\circ\ 30'$ — — 9.99799

To $EC = 131$ — — — — 2.11746

5. In the Triangle ABE

$$\text{Are given}\left\{\begin{array}{l} AB=53.25 \\ BE=107.4 \\ \angle E=13^\circ\ 30' \\ \angle A=28^\circ\ 5' \\ \angle B=138^\circ\ 25' \text{ by Axiom 10.}\end{array}\right\}\text{to find } AE.$$

As Sine $\angle E = 13^\circ\ 30'$ — — 9.36818

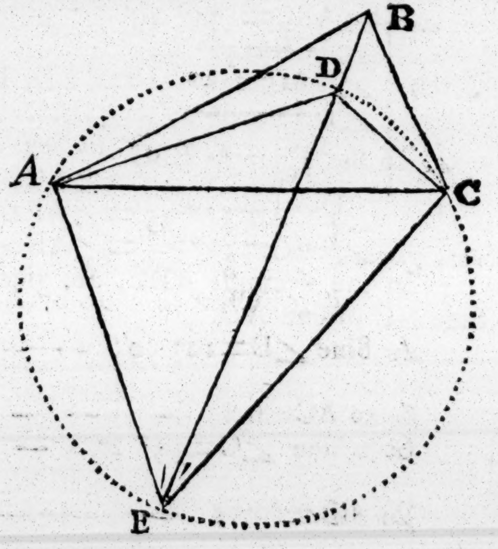
Is to $AB = 53.25$ — — — — 1.72631
So is Sine $\angle B = 138^\circ\ 25'$ — — 9.82197

To $AE = 151.4$ — — — — 2.18010

PROB. V. *Caſe* II.

Suppoſe the three Objects A, B, C were ſeen from E, and ſtood as ꝑ Figure, whoſe Diſtances are $AC = 12$ Furlongs, $AB = 9$ Furlongs, and $BC = 6$ Furlongs, the Angle $AEB = 33^\circ\ 45'$, and $CEB = 22^\circ\ 30'$, it is required to determine how far E is from A, B, C?

First make a Triangle ABC, with the three given Sides, draw CD ſo as to make an Angle with $AC = \angle AEB = 33^\circ\ 45'$, and at A draw AD ſo as to make an Angle with $AC = \angle CEB = 22^\circ\ 30'$, through the three Points A, C, E deſcribe a Circle, produce BD to the ſtation Place at



E, and join AE, CE, then EA, EB, EC are the Lines required by the Queſtion.

1. In the Triangle ABC, all the Sides are given to find the Angles. By Ax. 6.

To find $\angle A$.

6
12
9
Sum $=27$

Half Sum $=13.5$ Co. Ar. — — — 8.86967
Remainders { 7.5 Co. Ar. — — — 9.12494
Remainders { 1.5 Log. — — — — 0.17609
Remainders { 4.5 Log. — — — 0.65331

18.82391

Tang. $14^\circ : 28\frac{1}{2}'$ 9.41195
2

$\angle A=28 : 57$

By the ſame Method you will find $\angle C=46^\circ\ 34'$, and $\angle D=104^\circ\ 29'$, whence there are

Given {
$\angle BAC=28^\circ\ 57'$
$\angle ABC=104^\circ\ 29'$
$\angle ACB=46^\circ\ 34'$
$\angle DAC=22^\circ\ 30'=CED$, by Cor. Th. 20.
$\angle DAB=6^\circ\ 27'=\angle BAC-\angle DAC$.
$\angle ADC=123^\circ\ 45'$ by Ax. 10.
$\angle DCB=12^\circ\ 49'=\angle ACB-\angle ACD$.
}
$\angle DCA=33^\circ\ 45'=\angle AED$, by Cor. Th. 20.
$\angle AEC=56^\circ\ 15'=\angle AEB+\angle BEC$.

2. In the Triangle ADC, are given all the Angles and Side AC to find AD.

As Sine $\angle D=123^\circ\ 45'$ — — 9.91984

Is to $AC=12$ — — — — 1.07918
So is Sine $\angle C=33^\circ\ 45'$ — — 9.74477

To $AD=8.018$ — — — 0.90407

3. In the Triangle ADB

$$\text{Are given}\left\{\begin{array}{l}AB=9\\AD=8.018\\\angle DAB=6^\circ\ 27'\end{array}\right\}\text{to find the Angles D \& B.}$$

As Sum Sides 17.018 — — — 1.23090

Is to their Diff. .982 — — — $\bar{1}.99211$

So is Tang of half ∠∠'s D & B=86° 46' 11.24801

To Tang. half diff.=45° : 37' — 10.00922

From 86 : 46

∠ABE=41 : 09

$\angle ADE=47^\circ\ 36'=\angle ABE+\angle DAB=\angle ACE.$

4. In the Triangle ABE

$$\text{Are given}\left\{\begin{array}{l}\angle AED=33^\circ:45'\\\angle ABE=41\ :\ 09\\\angle BAE=105:\ 6\\\quad AB=9\end{array}\right\}\text{to find AE and EB.}$$

As Sine ∠AEB=33° 45' — — 9.74473

Is to AB=9 — — — — — 0.95424

So is Sine ABE=41° 9' — — 9.81824

To AE=10.6 — — — — 1.02775

As Sine ∠AEB=33° 45' — — 9.74473

Is to AB=9 — — — — 0.95424

So is ∠BAE=105° 6' — — — 9.98474

To EB=15.64 — — — — 1.19425

5. In the Triangle EAC

$$\text{Are given}\left\{\begin{array}{l}\angle CAE=76^\circ\ 9'=\angle BAE-\angle BAC.\\\angle AEC=56^\circ\ 15'\\\quad AC=12\qquad\text{to find EC.}\end{array}\right.$$

As Sine $\angle E = 56^\circ\ 15'$ — — 9.91984

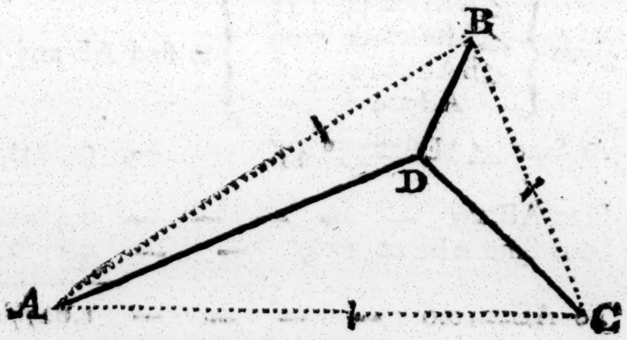
Is to $AC = 12$ — — — — 1.07918
So is Sine $\angle A = 76^\circ\ 9'$ — — — 9.98718

To $EC = 14.01$ — — — — 1.14652

Hence $AE = 10.6$
$EB = 15.64$
$EC = 14.01$

P R O B. V. *Caſe* III.

Let us again ſuppoſe the Place of our Station was at D, ſomewhere within the Triangle of the three Objects, as in the under Figure, whoſe Diſtances from each other are as in the laſt Problem, where being at D, I obſerved



Given The $\angle DAC = 22^\circ\ 30'$
$\angle ACD = 33^\circ\ 45'$
$\angle ADC = 123^\circ\ 45'$ by Ax. 10.
And $AC = 12$ to find AD, DC.

As Sine $\angle ADC = 123^\circ\ 45'$ — — 9.91984

Is to $AC = 12$ — — — — 1.07918
So is Sine $\angle DAC = 22^\circ\ 30'$ — — 9.58283

To $DC = 5.523$ — — — — 0.74217

As the Sine ∠ADC=123° 45′ — 9.91984

Is to AC=12 — — — — 1.07918
So is Sine ∠ACD=33° 45′ — — 9.74473

To AD=8.01 — — — — 0.90407

Given two Sides and the contained Angle in the Triangle ADB, to find DB.

As Sum Sides 17.01 — — — 1.23070

Is to their Diff. 1.01 — — — 0.00432
So is Tang. half Sum ∠∠'s=86° 46′ 11.24801

To Tang. half diff.=46 : 26 10.02163

Remains ∠ABD=40 : 20 and BAD=6° 27′=∠BAC—∠DAC.

As Sine ∠ABD=40° 20′ — — 9.81106

Is to AD=8.01 — — — — 0.90407
So is Sine ∠BAD=6° 27′ — — 9.05051

To BD=1.392 — — — — 0.14352

Hence AD=8.01, DC=5.523, and DB=1.392.

I have been more particular in the Solution of this Problem ſince the ſeveral Varieties of it ariſe from the different Poſitions of the Station E, than any of the foregoing; becauſe this Problem is of very great uſe for drawing Maps of Countries, or laying down Rocks, Sholes, or Sands, in Sea Charts, as it is done at one Station, or by one ſingle Obſervation.

PROBLEM VI.

Let Figure I. repreſent two Baſtions of a Fortification, with a Raveline, and it is required to plant two Batteries to play on the Faces AH, BF of thoſe Baſtions, to determine the proper Diſtances where the Batteries muſt be planted.

Suppoſe at ſome convenient Diſtances, as at R and S, I pitch upon for the two Places of my Station, whoſe Diſtance I meaſure ſuppoſe 155 Paces, and at R and S, I take the following Angles.

∠ARB=47° : 17′
∠BRS=46 : 50
∠ASB=58 : 24
∠ASR=39 : 16

To find the Diſtance of each Battery R and S, from the ſaliant Angles A and B.

1. ∠ARB+∠BRS=∠ARS=94° 7′ and ∠RAS=46° 37′ by Ax. 10.

2. ∠ASR+∠ASB=∠BSR=97° 40′ and ∠RBS=35° 30′ by Ax. 10.

1. In the Triangle ARS are given all the Angles, and Side RS, to find AR and AS.

As Sine ∠RAS=46° 37′ — —	9.86140
To RS=155 — — — —	2.19033
So is Sine ∠ARS=94° 7′ — —	9.99888
To AS=212.7 — — — —	2.32781
As ∠ARS=94° 7′ — — —	9.99888
To AS=212.7 — — — —	2.32781
So is Sine ∠ASR=39° 16′ — —	9.80136
To AR=135 — — — — —	2.13029

2. In the Triangle BSR all the Angles, and Side RS, are given, to find BR and SR.

As Sine ∠SBR=35° 30′ — —	9.76395
To SR=155 — — — —	2.19033
So is Sine ∠BRS=46° 50′— —	9.86295
To SB=194.7 — — — — —	2.28933

As Sine ∠RBS=35° 30′ — — 9.76395

To RS=155 — — — — 2.19033
So is Sine ∠RSB=97° 40′ — — 9.99610

To BR=264.5 — — — — 2.42248

PROBLEM VII.

To find the Diſtance of any Place from you by the Motion of Sound, from the firing of Cannon.

Sound, it is found from Experiments, flies after the Rate of 1142 Feet in a Second of Time; a meaſured Mile is 5280 Feet, or 1760 Yards, or 1056 Paces, or 880 Fathoms, and therefore a Sound requires 4.62 Seconds, or 9.24 half Seconds to fly a Mile.

Hence the Time that a Sound is going between any two Places being known, the Diſtance between thoſe two Places may eaſily be had.

For if you obſerve the Time between firſt ſeeing the Exploſion and hearing the Report of the Gun, the Diſtance of the Place from the Obſerver, where the Gun was fired, is eaſily had thus,

As 4″.62 to 1 Mile, ſo is the given Time between ſeeing the Exploſion and hearing the Report, to the Diſtance in Miles required.

Example I.

An Engineer was ordered to plant a Battery to fire on the Enemies, and obſerving by their firing ſome Guns, he counted eleven Seconds between the Time of his ſeeing the Flaſh and hearing the Report, how far was the Engineer from the Enemies Guns?

4.62) 11.000 (2.4 Miles nearly.
9 24

1760

Or rather thus,

As 4′.62 is to 1056 Paces, ſo is 11 Seconds to 2514 Paces, the Diſtance required.

Example

Example II.

Being one Day ordered to obſerve how far a Battery of Cannon was from me, I counted by my Watch 17 Seconds between the Time of ſeeing the Flaſh and hearing the Report; how far was the Battery from me?

As $4''.62 : 5280$ Feet : : $17'' : 19428$ Feet, or about $3\frac{1}{2}$ Miles.

Or you may come very near the Truth by counting the Pulſations at the Wriſt, for it is obſerved there are 75 Pulſations in a Minute, or about 19 in a Quarter of a Minute or 15 Seconds, when the Perſon is in good Health and a regular Habit of Body, and at the ſame Time free from any hurry or flutter in his Spirits.

Hence in 6 Beats of the Pulſe Sound flies about a Mile.

Example.

On ſeeing the Flaſh of a Gun I counted 21 Pulſations at the Wriſt, before I heard the Report; how far was the Gun from me?

6)21($3\frac{1}{2}$ Miles. *Anſwer.*

Explanation of Figure I.

NMAHG, EFBKL repreſent two Baſtions.
EF, GH the Flanks }
FB, AH the Faces } of thoſe Baſtions.
AB the outer Side of the Polygon.
DC the inner Side.
AP the outer Radius.
DP the inner Radius.
AD the Capital.
GE the Curtain.
DG the Demi-Gorge.
GN the Gorge.
∠APB the Angle of the Center.
∠HGE the Angle of the Curtain, or Angle of the Flank.
∠GHA the Shoulder Angle.
∠FBK the Angle of the Baſtion, or flanked Angle.
abc a Ravelin.

A

A *Rampart* is a Body of Earth raiſed about a Town to diſcover an Enemy and fire on them, they are made to cover the Buildings from thoſe in the Field, and conſiſt of ſeveral Baſtions and Curtains; the Soldiers continually keep guard here, and Pieces of Artillery are planted for the Place's Defence.

The *Parapet* is an Elevation of Earth and Stone raiſed upon the Edge of the Rampart towards the Field, to cover the Beſieged. Behind it the Soldiers ſtand ſecure. On the upper Part it is ſloped, which is called the *Glacis:* It has Embraſures and Merlons.

The *Ditch* or *Moat*, is a Channel made about a Place to avoid Surprizes. The Edge of the Ditch towards the Town is called the *Scarp*, and that towards the Field is called the *Counterſcarp;* all Moats muſt be well flanked, and muſt be made wider than any Ladder, Tree, *&c.* can reach acroſs them.

The *Covered Way* is a Place left on the Counterſcarp 24 or 30 Feet broad, which is always round the Moat next the Counterſcarp, and is covered on the Side next the Country by a Parapet or Bank; it is often very difficult to make a Lodgment on the Covered Way, becauſe the Beſieged generally Palliſado it along the middle.

Line of Circumvallation is a Trench or Line made by the Beſiegers within Cannon Shot of the Place they are going to beſiege, it ranges round their Camp, and ſecures its Quarters againſt the Relief of the Beſieged.

A *Battery* is a Place raiſed a little on Purpoſe to plant Cannon on to play upon the Enemy, fixed on Platforms of ſtrong Planks to ſupport the Wheels of the Carriages that they may not ſink in the Ground; the open Spaces in the Battery are called *Embraſures*, through which the Muzzles of the Cannon are put, and the Diſtances between the Embraſures are called *Merlons*, which are moſtly 9 Feet wide within, and 6 without, or the Cannon are planted about 15 Feet diſtant from one another, that the Parapet may be ſtrong, and the Gunners have room to work.

A *Cavalier* is a Heap of Earth raiſed in a Fortreſs to lodge the Cannon, for ſcouring the Field; they are ſometimes of a round, and ſometimes of a ſquare Figure, and always have a Parapet to cover the Cannon there ſet in

Battery to defend the Face of the oppoſite Baſtion, and the Baſtion itſelf, if the Enemy makes a Lodgment in it. There muſt be about 14 Feet between Cannon and Cannon. They are often made upon the middle of the Curtain near the Parapet, to ſee the Field from the Place, to diſcover the Enemy in his Works, and to double the Fire of the defended Town in ſuch Places as may be attacked.

A *Redoubt* is a ſmall Fort of a ſquare Figure, in order to diſturb the Enemies Works beyond the Glacis, and are built within Muſket Shot of the Outworks of the Place, to defend it.

I ſhall ſay no more of Fortification, it being a peculiar Branch of Science of itſelf, but make a Tranſition to Gunnery. But firſt of Motion.

Velocity is that Affection of Motion whereby a Body runs a given Space in a given Time, whence, from the Principles of Mechanics,

$$\text{Velocity} = \frac{\text{Space}}{\text{Time}}$$

Space = Velocity multiplied by the Time.

$$\text{Time} = \frac{\text{Space}}{\text{Velocity.}}$$

The *Momentum* which conduces to the Effecting of Motion, is compounded of the Velocity and quantity of Matter, or Weight, whence the *Momentum*=*Velocity* multiplied by the *Weight*.

$$\textit{Weight} = \frac{\text{Momentum}}{\text{Velocity}}$$

$$\textit{Velocity} = \frac{\text{Momentum}}{\text{Weight.}}$$

GUNNERY.

GUNNERY.

GUNNERY is the Application of the Doctrine of Projectiles, and is a Science highly neceſſary to a martial People; it ſhews the Properties, Uſe and Compoſition of Gun-Powder, gives the different Sizes, Figures, Charges, Carriages of Cannon, Mortars, Pedreros, *&c.* how to point, prove and manage them; it ſhews the Art of throwing Balls, Bombs, Carcaſſes, Cartouches, Granadoes, *&c.* the manner of nailing up Cannon, ſo as to render them uſeleſs to an Enemy, and widening the Touch-hole; fixes Rules for the making of Batteries, Pontons, Bridges, and conſtructing Mines, Countermines, *&c.* and all thoſe Engines and Fireworks neceſſary for taking and defending fortified Towns.

It is neceſſary for an able Gunner to be well acquainted with Arithmetic, to keep an accurate Account of all remarkable Shots, to be ready at the finding the requiſite Charges of Powder fitting for any Piece required; the Weight of all Sorts of Shot; to be acquainted with the Extraction of Roots, to underſtand ſo much of Geometry and Trigonometry as to be expeditious in taking Heights and Diſtances, to level, conſtruct Batteries and Platforms, to know the Names and different Parts of Cannon, their different Ranges, the Powder for Proof and Service, the Point Blank on a Level, utmoſt Random; Horſes or Men to draw the Pieces on ſudden Emergencies, the beſt Method for the Arangement of Artillery, according to the Nature and Circumſtances of the Place; to know the Nature and Uſe of the Fuſes for Bombs, and the neceſſary Inſtruments relating to Cannon, to be expert in conſtructing Mines and their Galleries, the moſt uſeful Fireworks for incommoding an Enemy, ſuch as Bombs, Granado Balls, Powder

Sacks, Torches, thundering Hedghogs, thundering Barrels, tarred Faggots, *&c.*

DEFINITIONS.

I. Angle of $\left\{\begin{matrix}\text{Elevation}\\ \text{Depreſſion}\end{matrix}\right\}$ according as it is $\left\{\begin{matrix}\text{above}\\ \text{below}\end{matrix}\right\}$ the horizontal Line, is the Angle which the Line of Direction makes with the Horizon.

II. The $\left\{\begin{matrix}\text{Amplitude}\\ \text{or Random}\end{matrix}\right\}$ is the Diſtance between the Point of Projection and the Object aimed at.

III. The *Impetus* is the greateſt Height that any Cannon Ball can aſcend, when projected perpendicularly to the Horizon; and is always juſt one half of the greateſt Random upon a horizontal Plain.

The greateſt Random muſt be found by Trials, which is neceſſary to be known.

IV. When any Body is projected in any Direction whatever, which is not perpendicular to the Plain of the Horizon; it will in its Motion deſcribe a Curve Line, called a Parabola, unleſs ſo far as the Reſiſtance of the Air hinders it, the Curve A E B is the Parabola. *Fig.* II.

V. If the Body projected meet with no Reſiſtance, it would throw it in the Direction A C, but Gravity acts every Inſtant, and draws the Body from the Direction A C, as ſoon as it leaves the Engine, in the Curve A B.

VI. The Direction A C, of any Engine, is a Tangent at the Point where the Engine is placed, to the Curve A E B, which the projected Body deſcribes by its Flight; for the Curve A E B is deſcribed by the projected Body in the ſame Time that it would have fallen through its height C B.

VII. If A C is the Velocity and Direction of a projected Body, then A B is the horizontal Velocity.

VIII. B A C is the Angle of Direction, or the Angle for elevating your Piece, and the greater the Elevation is the higher the Ball will riſe; whence a Body thrown up with a given Velocity from A, the Place where your Piece is fixed, will riſe to a Height equal to a fourth Part of the Parameter belonging to that Point.

IX. The greateſt Random is half the Parameter, and when the utmoſt or greateſt Random is mentioned, it muſt be underſtood that the Piece is always made at the Elevation of 45 Degrees, twice the Sine whereof is Radius; and likewiſe Ranges made equally diſtant above and below 45°, are equal; for the Sines of all double Arches are equal to the Sines of their double Complements.

X When any Object's Diſtance on a horizontal Plain, is leſs than the Piece's greateſt Random, it may be hit then by two Elevations, each whereof is the Complement of the other to 90 Degrees.

XI. Since the verſed Sine of 180 Degrees is equal to twice the Radius, it is manifeſt, that the greateſt Altitude of a perpendicular Projection is one fourth of the Parameter, or half of the greateſt horizontal Random, and conſequently, the Impetus is half of the greateſt horizontal Random.

XII. The Velocity of the Impulſe moved in a Second, is always a mean Proportional between the Parameter and $16\frac{1}{12}$ Feet; that is, equal to the Square Root of the Product of $16\frac{1}{12}$ Feet into the Parameter.

XIII. The Velocity of a Project in any Point of its Curve-Line made by its Flight, is ſuch, as a Body acquires in falling down a fourth Part of the Parameter belonging to that Point.

XIV. In Projects, caſt with the ſame Velocity, if the Velocity be defined by the Number of Feet moved in a Second, the Parameter will be found by dividing the Square of the Velocity by $16\frac{1}{12}$ Feet.

XV. The utmoſt Random on an inclined Plain, and conſequently, the leaſt Force that can reach an Object, is made when the Axis of the Piece makes equal Angles with the Zenith and the Object; ſo that there needs no more to be done, but to let the Axis of the Piece, properly charged, biſect the Angle between the Object and Zenith; and the Projections are equal at Elevations equally diſtant from this Line above and below; and this is true, whether the Projections be made on Plains above the Level of your Piece, or below it.

XVI. The general Rule for hitting any Object, is to get your Diſtance from it, as near as you poſſibly can, the Impetus of your Piece you muſt know from Experiments, then

As twice the Impetus is to Radius,

So is the Diſtance of the Object from you, to the Sine of double the Angle of Elevation, half whereof is the Angle of Elevation required.

XVII. When any Object is ſo near you as you can take Aim with your Eye, as you may always in firing Point Blank, and in battering Walls; then *diſpart* your Piece, that is, ſet ſuch a Mark as a Piece of Wood hollowed on one Side to fix on the Cannon's Convexity, on or near the muzzle Ring, that a Sight-Line taken on the Top of the Baſe Ring againſt the Touch-Hole, by the Mark ſet at or near the Muzzle, may be parallel to the Cannon's Cylinder, its Top being juſt as high above the Piece as the Metal is thicker at the Breech than at the Mouth, this *Sight-Piece* ſerves for guiding your Eye in taking Aim, and by levelling the upper Part of the Breech and the upper Part of the Sight, with a given Object, the Bullet diſcharged in that Direction will be carried towards that Object, only lower juſt half the Diameter of the Cannon at its Breech, therefore always level the Piece half the Diameter of the Breech higher, the Ball then will hit the Object; provided no Accident vary the Direction.

XVIII. Cannon are beſt named from the Weight of the Ball they carry;

As a Cannon carrying a Ball weighing
30 *lb.* is a thirty Pounder.
24 *lb.* is a twenty four Pounder.
16 *lb.* is a ſixteen Pounder.
&c. *&c.*

XIX. M. *Du Metz*, Lieutenant General of the late *French* King's Forces, and Lieutenant of the Artillery in *Flanders*, obſerved by which the Pieces being fired at random Shot, and charged as follows, it appeared that

		Fathoms.
A 24 Pounder charged with	16*lb.* of Powder, carried	2250
16 Pounder — —	$10\frac{1}{2}$*lb.* — — —	2020
12 Pounder — —	8 *lb.* — — —	1870
8 Pounder — —	$5\frac{1}{4}$*lb* — — —	1660
4 Pounder — —	$2\frac{3}{4}$*lb.* — — —	1520

But notwithſtanding, with what Care ſoever the above Experiments were made, many Accidents will occur ſo as to render the Diſtance of the Shot uncertain; and though not ſo abſolutely to be depended on, yet ſo much regard is to be paid to theſe Experiments, as they give pretty nearly the true Extent of the Shot.

XX. M. *de Vauban* was a great Improver in the practical Management of Artillery, eſpecially that called by him *Batterie à ricochet*, which he fired with ſmall Quantities of Powder, and elevated the Piece ſo that the Ball in its Deſcent might juſt go clear of the Parapet of the Enemy, and drop into their Works; by this means, the Bullet coming to the Ground in a ſmall Angle, and conſequently with a ſmall Velocity, it either bounds or rolls along in the Direction it was fired in; and therefore if the Piece be placed in a Line with the Battery it is intended to ſilence, or the Front it is to ſweep, each Shot rakes the whole Length of the Battery or Front, and has infinitely more Chance of diſabling the Defendants, diſmounting their Cannon, killing and laming all it meets with, and creating great Diſorder.

XXI. A twenty four Pounder has been fired five or ſix times in an Hour for twenty four Hours together, but great Care muſt be taken to cool it, for the Piece will grow ſofter, and the Touch-Hole will be widened; the hotter a Cannon grows, the greater is the Velocity of the Ball, and it recoils with greater Violence upon the Carriage, becauſe the Powder takes fire quicker and in a greater Quantity; yet it may happen to ſhorten the Random of the Ball, if the Cannon recoiling falls again before the Bullet is out. Therefore the Platform muſt be firm and ſolid, for if the Platform be ſhaken with the firſt Impulſe of Powder, it is ſure the Piece will likewiſe be ſhaken, and this will certainly alter its Direction, and ſo render the Shot uncertain: In order to prevent this, Platforms are uſually made very firm to a conſiderable Depth backwards, ſo that the Piece is well ſupported, both in the beginning of its Motion and through a great Part of its Recoil; and Mr *Robins* has computed from Experiments, that a Piece ten Feet long, carrying a twenty four Pounder, charged with ſixteen Pounds of Powder, the Bullet will be out of the Piece before it has recoiled half an Inch.

XXII. When Cannon cannot very readily be taken away, or to render them uſeleſs to an Enemy, the uſual Way is to nail them up, which is driving a large Nail into the Touch-Hole, and leaving no Part of it to be taken hold on to draw it out, and this renders them quite uſeleſs; Pebbles or any hard Stones will do, if Nails are not at Hand.

XXIII. Red-Hot Bullets are often made uſe of to fire Towns, *&c.* theſe ſort of Bullets are made Flame or Red-Hot on a Grate contrived on purpoſe, and then put into the Mortar, or Piece, with a Pair of Tongs, the Piece being wadded before with Turf; for the Powder muſt be covered with ſomething of this nature, or elſe the hot Bullet would ſet fire to the Powder, and render the Force ineffectual; then as ſoon as the Bullet is arrived at the Wad of Turf, the Piece is immediately fired, for it cannot well be rammed down with Wad, for fear the Powder in the mean time ſhould take fire and do Miſchief. Seldom heavier than ſix, eight or ten Pound Bullets are uſed this way, for they would be troubleſome to manage.

XXIV. There is another Engine of Metal made uſe of on particular Occaſions, called a *Petard*, pretty much reſembling a high crown Hat, with narrow Brims; this is filled with very fine Powder, the Engine is fixed to a Plank, bound faſt down with Ropes going thro' Handles, which are round the Rim of its Mouth; it is well primed: its uſe is to break down Gates, Drawbridges, Portcullices, Barriers, *&c.* it alſo is for breaking through the Enemies Galleries, and to fruſtrate their Mines; the Officers who are to manage the Petard, when made uſe of, are in a dangerous Poſt, few ever returning ſafe from the Expedition, if we may credit M. *St Remy*, the Garriſon inceſſantly firing on them.

XXV. Bombs are hollow Balls of caſt Iron, filled with Powder, old Nails, Pieces of Iron, *&c.* they are deſigned rather for firing Houſes, Magazines, annoying a Garriſon, than any Execution by their projecting Force; being ſhot out of Mortars, ſome of the larger ſort weigh four or five hundred Weight. In adjuſting the Fuſes of Bombs, I have inſerted at the Concluſion of this Work ſome Problems in order to proportionate them to the time of their Flight. The Fuſes of Bombs or Granado Shells, are they which make the whole Powder or Compoſition in the Shell take fire, to do the deſigned Execution; they are round, and thicker at one end than the other, filled with a Compoſition of Sulphur, Salt Petre, and the beſt Powder, and is deſigned to burn ſo long and no longer, as is the time of the Motion of the Bomb from the Mouth of the Mortar to the Place where it is to fall; ſo that the Fuſe muſt be contrived either from the Nature of the Wildfire, or the Length of the Pipe that contains it, to burn juſt that time.

XXVI. When the Bomb is charged, the Fuſe is put into it through the Touch-Hole, and after it is fired it ſets fire to the Powder in the Bomb; the Fuſes muſt be charged with great Care for anſwering the End they are deſigned, that nothing may prevent their firing the Powder in the middle of the Bomb; and as they are generally charged long before they are uſed, the Ends are well ſecured, leſt the Compoſition get wet, or fall out; and when they are uſed the ſmall End is opened or cut off, and put into the Bomb, but the bigger End is never opened till the Bomb be in the Mortar, and juſt a going to be fired.

XXVII. The lower Part of the Shell is the thickeſt Part of it, which is neceſſary it ſhould be ſo, that being the heavier, it may always fall upon it, and never on the Fuſe; if the Bomb be ſet fire to by the Fuſe before it reaches the deſigned Place, it will burſt in the Air, and then it may be as fatal to the Perſons that fired the Mortar, as thoſe againſt whom it was intended; and to remedy this Inconvenience, the Fuſe is made to laſt as long as the Bomb is going its utmoſt Random, if thrown at a great Diſtance; but when near, part of the Fuſe is burned out before diſcharged.

XXVIII. There are Pieces called *Hawitzers*, which are about eight Inches in the Bore, and above three Feet long; they are uſed to throw Bombs into the Platform of a Baſtion, or the middle of a Parcel of Men.

XXIX. The beſt battering Pieces are moſtly twenty four Pounders, they being found abundantly ſufficient for making Breaches; moreover they are better manageable, and take leſs Ammunition than Cannons that are larger. The modern way of making Breaches, is firſt cutting off the whole Wall as low as poſſible, before its upper Part is attempted to be beat down.

XXX. Balls ſtrike with a greater Force againſt rough Stones than ſmooth ones, and the greater the Velocity the greater the Force, for the Weight of the Ball in Pounds multiplied by the Velocity, gives the Force of the Ball at that Place where the Velocity was taken; as for inſtance, if a Piece carrying a 24 *lb* Ball, and its Velocity be 500 Feet in a Second, when it arrives near an Object it is deſigned to hit, the Force of this Ball will then be 12000*lb* or 5 Ton, 7 Hundred, and 16 Pounds.

XXXI. Small Bodies with great Velocity, are more proper to tear in Pieces; and great Bodies with ſmall Velocity, to ſhake or move the Whole.

XXXII. That Part of a Cannon next us when it ſtands ready to fire, is called by the Artillery People the *Breech* or *Coyle*; the round Knob at the End of it is called the *Caſcabel:* from this End I ſhall here ſet down the ſeveral Parts of a Cannon.

1. The Caſcabel.
2. The Caſcabel Aſtragal.
3. The Neck of the Caſcabel.
4. The Breech and Breech Mouldings, is all the Metal behind the Touch-Hole.
5. The Baſe Ring and Reſiſtance.
6. The Vent and Vent Aſtragal, is the next Corniſh or Ring to the Vent
7. The Charging Cylinder.
8. The firſt reinforc'd Ring, is that Corniſh next the Vent Aſtragal.
9. The firſt reinforc'd Aſtragal.
10. The vacant Cylinder.
11. The Dolphins.
12. The Trunnions are two Knobs of Metal by which the Cannon hangs in the Carriage.
13. The ſecond reinforc'd Ring.
14. The ſecond reinforc'd Aſtragal.
15. The Chace.
16. The Neck Aſtragal.
17. The Neck and Muzzle Ring.
18. The Muzzle Mouldings or Freeze.
19. The Face of the Piece.
20. The Bore or hollow Part.

XXXIII. I need not ſay any thing with regard how to make Cartridges, to fix the Hand Granades, Powder Cheſts, Powder Pots, Powder Tubs, &c. theſe are better learned in the Field: but proceed to the Solution of ſome of the moſt uſeful Problems in Practical Gunnery.

PROJEC-

PROJECTIONS on the Plane of the Horizon.

PROBLEM I.

Given the utmoſt Random of any Piece of Artillery, to find the Angle of Elevation to hit an Object at a known Diſtance.

RULE I.

As the Impetus (or half the greateſt Random) is to half the given Diſtance of the Object from you, ſo is Radius to the Sine of double the Angles of Elevation. Or,

RULE II.

As the greateſt Random is to the Object's given Diſtance, ſo is Radius to the Sine of double the Angles of Elevation.

Example.

Suppoſe the greateſt Random of a Cannon was 6840 Feet, how muſt the ſaid Piece be elevated to hit an Object a Mile off.

As Impetus 3420 Feet — — —	3.53402
Is to half a Mile=2640 Feet —	3.42160
So is Radius — — — —	10.00000
To Sine of 50° 31′ or 129° 29′ —	9.88758

Therefore $\frac{50° : 31'}{2} = 25° : 15'$ the Angle of the lower Elevation.

And $\frac{129° : 29'}{2} = 64° : 44'$ the Angle of the higher Elevation.

PROBLEM II.

Given the greateſt Random of any Piece, to find what Random the ſaid Piece makes at any other given Elevation.

RULE.

As Radius to the Sine of double the given Angle of Elevation, ſo is the greateſt Random to the Random required.

Example.

A Piece whoſe greateſt Random is known to be 6840 Feet, and the Engineer found that by elevating his Piece 25 15½′ he ſtruck an Object on the Plane of the Horizon, pray what Diſtance was the Object from him?

As Radius — — — — —	10.00000
To Sine of twice 25° 15½′ — —	9.88751
So is 6840 Feet — — — —	3.83505
To 5280 Feet, the Diſtance required	3.72256

PROBLEM III.

By knowing the Diſtance of any Object from you, and the Piece's Elevation, to find the Impetus, to hit that Object.

RULE.

As the Sine of twice the Angle of Elevation is to Radius, ſo is the given Diſtance of the Object to the greateſt Amplitude, half whereof is the Impetus required.

Example.

Suppoſe an Object a Mile diſtant from me, and by elevating the Piece 64° 44½′ I can ſtrike that Object, required the Impetus?

Twice 64° 44½′ is 129° 29′ its Sup. to 180° is=50° 31′.

As Sine of 50° 31′ — — — —	9.88751
Is to Radius — — — —	10.00000
So is 5820 Feet (1 Mile) — —	3.72263
To 6840 — — — — —	3.83512

Therefore $\frac{6840}{2}$=3420 Feet, the Impetus required.

PROBLEM IV.

Given the Angle at the lower Elevation, and the Object's Diſtance from you, to find the Altitude of the Projection.

RULE.

RULE.

As Radius to the Tangent of the lower Elevation, ſo is a fourth Part of the Object's Diſtance, to the Ball's Altitude required.

Example.

Suppoſe at 25° 15′ Elevation, I can hit an Object a Mile off, what height does the Ball fly at?

As Radius — — — — —	10.00000
To Tangent of 25° 15′ — —	9.67360
So is 1320 Feet — — — —	3 12057
To 623 Feet nearly, the Alt. required	2.79417

PROBLEM V.

Given the Altitude of Projection, and the Piece's Impetus, to find how far an Object is from you.

RULE.

From the Impetus ſubtract the Ball's Altitude, multiply this Remainder by the Altitude, the ſquare Root of the Product is a fourth Part of the Diſtance required.

Example.

Suppoſe the Height of a Ball projected be 623 Feet, and the Piece's Impetus 3420 Feet, required the Diſtance of the Object?

From 3420
take 623

2797
623

8391
5594
16782

1742531 whoſe Square Root is 1320 Feet, and 1320×4=5280 the Diſtance required.

PRO-

PROBLEM VI.

The Diſtance of an Object and the Ball's Altitude being given, to find the Piece's Elevations.

RULE.

As half the given Diſtance is to twice the Ball's Height, ſo is Radius to the Tangent of the $\left\{\begin{matrix}\text{lower}\\ \text{higher}\end{matrix}\right\}$ Elevation.

Example.

Suppoſe an Object a Mile diſtant from me, and the Ball's Altitude is 623 Feet, what are the Angles of Elevation?

As 2640 Feet — — — — 3.42160

Is to 1246 — — — — — 3.09551
So is Radius — — — — 10.00000

To Tangent 25° 16′ lower Elevation 9.67391
And 64° 44′ is the higher Direction.

PROBLEM VII.

Given the Angle of Elevation, together with its Amplitude, to find the Amplitude at any other given Elevation.

RULE.

As the Sine of twice the firſt Elevation, is to its given Amplitude, ſo is the Sine of twice the other Elevation, to the reſpective Amplitude required.

Example.

Suppoſe at an Elevation of 30° 20′ the given Diſtance be 2700 Feet, what Diſtance will the ſaid Piece throw a Ball, which is elevated at 39° 48′?

Twice 30° 20′=60° 40′, and twice 39° 48′=79° 36′.
As Sine 60° 40′ — — — — 9.94040

To 2700 Feet — — — — 3.43136
So is Sine 79° 36′ — — — 9.99280

To 3046 Feet the Diſtance required 3.48376

If you know what Diſtance a given Charge of Powder will effect, and would know what Charge will be ſufficient to hit an Object at any given Diſtance, within your Cannon's reach, without altering the Elevation, obſerve the following

PROBLEM VIII.

The Charge of Powder, and the Diſtance of Projection made from that Charge, being known, to find what Charge of Powder will do to hit an Object, at a known Diſtance, without altering the Piece's Elevation.

RULE.

As the given Diſtance is to its reſpective Charge of Powder, ſo is the known Diſtance, to its reſpective Charge of Powder.

Example.

Suppoſe a 24 Pounder, charged with 14 Pounds of Powder, will throw a Ball 6840 Feet, what Quantity of Powder muſt the ſaid Piece be charged with, to throw the ſaid Shot ſo as to hit an Object 8900 Feet from you, without altering the Piece's Elevation?

given diſt. *lb.* *diſt.*
As 6840 : 14 : : 8900 : 18$\frac{2}{10}$ *lb.* required.

N. B. This is only true when the Velocity is ſuppoſed to be in the ſubduplicate Ratio of the Quantity of Powder, which is not ſtrictly true.

PROBLEM IX.

Given a Piece's Impetus, to find the Velocity of the Ball.

RULE.

Multiply the ſquare Root of the Impetus by 8, the Product is the Velocity the Ball moves through in a Second.

Example.

Suppoſe the Impetus of a Piece be 14000 Feet, what is the Velocity of the Ball at the Object on the Horizon?

The Square Root of 14000 is 118.3.
And 118.3
8

946.4 Feet the Ball moves through in a *Second.*

PROBLEM X.

Given the Piece's Elevation and the Object's Diſtance in Feet, to know how long time the Ball is going to that Object.

RULE.

As the Radius is to the Tangent of the Piece's Elevation, ſo is the given Diſtance of the Object to the Square of four times the required Number of Seconds; this laſt Product divide by 2, and find the Number anſwering to it in the Log. Tables, a fourth of which Number is the Time of flight in Seconds.

Example.

At an Elevation of 38° 20′ I can hit an Object a Mile and a half from me, required the Time of the Ball's flight?

As Radius — — — —	10.00000
To Tangent 38° 20′ — — —	9.89801
So is 7920 Feet (1½ Mile) — —	3.89872
To the Square of 4 times the N° of Seconds	3.79673

Now half of 3.79673 is 1.89836; the Number anſwering to it is 79.14 a fourth of which is 19¾ Seconds, the Time of Flight.

But as to the throwing of Bombs, the Elevation of the Mortar is commonly at 45°, then a fourth Part of the Square Root of the Diſtance in Feet, will give the Number of Seconds required.

PROBLEM XI.

Given the Impetus (or greateſt Random) of any Piece of Cannon, and her abſolute Force in Pounds, to find with what Force ſhe will ſtrike an Object, at a given Diſtance, whoſe Surface leans backwards any given Angle from the Perpendicular.

RULE.

As the greateſt Random is to Radius, ſo is the Object's given Diſtance to the Sine of double the Angle of Elevation.

And as Radius is to the Sine of the Angle at the Surface of the Object, ſo is the given Force to the Force required.

Example.

Admit a Piece's greateſt Random be 9000 Feet, and the abſolute Force of the Ball 6952 Pounds, with what Force will ſhe ſtrike an Object 6000 Feet diſtant from you, and whoſe Surface leans backwards from the Perpendicular 36° ?

As 9000	3.95424
To Radius	10.00000
So is 6000	3.77815
To Sine 41° 48′	9.82391

Whence $\frac{41^\circ\ 48'}{2}=20^\circ\ 54'$ the Elevation that the Ball ſtrikes the Plane of the Horizon at, and conſequently the Surface of the Object at an Angle of 74° 54′, for $90^\circ-36^\circ+20^\circ\ 54'=74^\circ\ 54'$.

To find the Force.

As Radius	10.00000
To Sine 74° 54′	9.98474
So is 6952 *lb.*	3.84210
To 6712 *lb.* the Force required	3.82684

PROBLEM XII.

Given a Piece's greateſt Random, and the requiſite Charge of Powder, to find what Charge of Powder will ſuffice to ſtrike an Object at right Angles at a given Diſtance, whoſe Surface reclines any given Number of Degrees.

RULE.

As the Sine of twice the Angle of Elevation, is to the given Object's Diſtance, ſo is Radius to twice the Impetus, and as the Square Root of twice the Object's Diſtance is to the Square Root of twice the Impetus now found, ſo is the given Charge of Powder, to the Charge required.

Example.

Admit a Cannon's greateſt Random with a Charge of 16 Pounds of Powder was obſerved to be 3 Miles, what Charge will be ſufficient to ſtrike an Object at right Angles, whoſe Surface reclines 36° at a Mile and half diſtance from the Piece?

It is plain from the Queſtion that the Elevation muſt be 36°, to ſtrike it at right Angles. Then,

As Sine of 72° — — — — 9.97820

Is to $1\frac{1}{2}$ Mile=7920 Feet — — 3.89872
So is Radius 90° — — — — 10.00000

To 8328 Feet, double Impetus — 3.92052

$$\text{As } \sqrt{15840} : \sqrt{8328} :: \overset{lb.}{16} : \overset{lb.}{11.6} \text{ the Charge.}$$

PROJECTIONS on ASCENTS.

PROBLEM XIII.

Given the greateſt Random of any Piece on the Plane of the Horizon, to find the greateſt Random of the ſaid Piece on a Plane of a given Aſcent.

RULE.

RULE.

To the natural Sine of the given Aſcent add Radius; then ſay, as that Sum is to Radius, ſo is the greateſt Random on the Plane of the Horizon to the greateſt Random required.

Example.

Suppoſe a Cannon's greateſt Random on the Plane of the Horizon be 7980 Feet, what is her greateſt Random on that Plane, which is 8° 40′ above the Level of the Piece?

Here the natural Sine of 8° 40′ is .1506, and Radius =1, then 1.1506 is the Sum, but 1.15 is near enough for Practice.

As 1.15 : 1 :: 7980 : 6939 Feet, the greateſt Random required.

PROBLEM XIV.

Given the greateſt Random on a Plane of a given Aſcent, to find the greateſt Random on the Plane of the Horizon.

RULE.

As Radius is to the Sum of the natural Sine of the Plane's Aſcent and Radius, ſo is the greateſt Random given, to the Random on the Plane of the Horizon required.

Example.

There is a Plane which is 8° 40′ above the Level of my Piece, the greateſt Random of my Piece on that Plane was obſerved to be 6939 Feet, required the greateſt Random of the ſaid Piece on the Plane of the Horizon?

As 1 : 1.15 :: 6939 : 7980 the Random required.

PROBLEM XV.

Given the Piece's Elevation and its horizontal Random for that Elevation, to find the Random on a Plane of a given Aſcent.

RULE.

From the natural Tangent of the Piece's Elevation, take the natural Tangent of the Plane's Elevation, and find the Angle correſponding to that Difference; then add that

Angle to the Plane's Elevation, and ſay, as the Sine of that Sum is to the given horizontal Random, ſo is the Angle correſponding to the Difference, to the Random required.

Example.

A Cannon's horizontal Random at an Elevation of 50° 6′ was found to be 5820 Feet, what is her Random at the ſame Elevation above the Horizon, on that Plane whoſe Aſcent is 10° 20′?

From natural Tangent 50° : 06′=1.19598
Take natural Tangent 10° : 20′=0.18233

The Angle correſponding is 45° : 23′=1.01365 Diff.

To 45° : 23′
Add 10° : 20′

Sum 55° : 43′

As Sine of 55° 43′ — — — 9.91711

Is to Log. 5820 — — — — 3.76492
So is Sine 45° 23′ — — — 9.85237

To 5014 the Random required — 3.70018

Corollary. Having the Angle of Obliquity of the Plane, you have the Elevation for the utmoſt Random of any Piece, by adding half the Angle of Obliquity to 45°.

PROBLEM XVI.

Given a Piece's Random at any given Elevation, on the Plane of the Horizon, to find her Random at another given Elevation, on the Plane of a given Aſcent.

RULE.

As the Sine of twice the given Random's Elevation, is to the given Random, ſo is twice the Sine of the other Elevation, to her horizontal Random.

Then from the natural Tangent of the other given Elevation, take the natural Tangent of the Plane's Aſcent, and ſee what Angle anſwers to their Difference, the Sum of theſe two taken from 180° leaves a third Angle.

As Sine of this third Angle is to the horizontal Random, ſo is the Sine anſwering to the Difference, to the Random required?

Example.

At an Elevation of 50° 6′ I obſerved a Cannon's Random on the Plane of the Horizon was 5820 Feet; pray what is her Random at 60° 6′ Elevation above the Horizon, on a Plane whoſe Aſcent is 10° 20′?

As twice Sine of 50° 6′ — — 9.99308

Is to 5820 Feet — — — — 3.76492
So is twice the Sine of 60° 6′ — 9.93665

To 5111 Feet, the horizontal Random 3.70849

From natural Tangent 60° : 06′=1.73905
Take natural Tangent of 10 : 20 = .18233

Remains ∠ — — 57 : 17=1.55672
Add 10 : 20

Take 67 : 37
From 180 : —

112° : 23′ its Sup.=67° 37′.

As Sine 67° 37′ — — — — 9.96598

Is to 5111 — — — — — — 3.70849
So is Sine 57° 17′ — — — 9.92497

To 4650 Feet, the Random required 3.66748

PROBLEM XVII.

Given the Angle of Direction, and the Random on the Plane of the Horizon, to find how high a Cannon can ſtrike an Object, on a Perpendicular, whoſe horizontal Diſtance is alſo known.

RULE.

RULE.

As the Tangent of the Angle of Direction is to Radius, ſo is the Random on the Plane of the Horizon to a *fourth Number*, and as this fourth Number is to the Object's horizontal Diſtance, ſo is the Difference of the Diſtances, to the Height required.

Example.

The horizontal Random of a Cannon at an Elevation of 50° 6′ was obſerved to be 5820 Feet, how high will the ſaid Cannon ſtrike an Object, with the ſame Direction on a Perpendicular, whoſe horizontal Diſtance is 5000 Feet?

As Tangent 50° 6′ — — — 10.07772
Is to Radius 90° — — — 10.00000
So is 5820 — — — — — 3.76492
To 4866 a fourth Number — — 3.68720

Then from 5820 take 5000, leaves 820, Difference of the Diſtances.

And 4866 : 5000 : : 820 : 843 Feet *ferè*, the Height required.

PROBLEM XVIII.

Given a Piece's Impetus, and the Diſtance of an Object upon a Plane, of a given Elevation, to find the Direction to hit that Object.

RULE.

As Radius is to the Object's Diſtance, ſo is the Sine of the Plane's Elevation, to the perpendicular Height of the Object; from twice the Impetus take the Object's Height, and note the *Remainder.*

To and from this Remainder add and ſubtract the Object's Diſtance, to find the Sum and Difference.

To the Logarithm of the Sum add the Logarithm of the Difference, and find the Number belonging to half this Sum.

Add and ſubtract that Number to and from twice the Impetus, and you will have a greater and a leſſer Number, then ſay

As the Height of the Object
To the $\left\{\begin{matrix}\text{leſſer}\\ \text{greater}\end{matrix}\right\}$ Number,
So Tangent, Plane's Elevation
To Tangent $\left\{\begin{matrix}\text{leſſer}\\ \text{greater}\end{matrix}\right\}$ Elevation above the Horizon.

Example.

Admit a Cannon's Impetus be 5820 Feet, required at what Directions muſt the Piece be elevated, ſo as to hit a Caſtle ſtanding upon a Hill, whoſe Height is 8° 30′ above the Level of the Piece, and direct Diſtance 7000 Feet?

As Radius — — — — — 10.00000

Is to 7000 — — — — — 3.84509
So is Sine of 8° 30′ — — — 9.16970

To Object's Height=1035 — — 3.01479

From twice Impetus=11640
Take Object's Height= 1035

Remainder=10605
Add Diſtance= 7000

Sum=17605

Difference= 3605

To Log. 17605 — 4.24563
Add Log. 3605 — 3.55690

Sum=7.80253

The Number to it is $7966\frac{1}{2}$=3.90126 Half Sum.

To twice the Impetus=11640
Add and ſubtract $7966\frac{1}{2}$

Greater Number=$19606\frac{1}{2}$ its Log.=4.29240

Leſſer Number= $3673\frac{1}{2}$ its Log.=3.56507

As 1035 — — — — —	3.01494
Is to $3673\frac{1}{2}$ — — — — —	3.56507
So is Tangent of 8° 30′ — —	9.17449
To Tangent of 27° 57′ — — The lower Elevation.	9.72462
And as 1035 — — — —	3.01494
Is to $19606\frac{1}{2}$ — — — —	4.29240
So is Tangent 8° 30′ — — —	9.17449
To 70° 33′ the higher Elevation= Above the Horizon.	10.45195

PROBLEM XIX.

Given the horizontal Diſtance, and the perpendicular Height of any Object, to find the leaſt Impetus that can poſſibly reach that Object.

RULE.

To the Square Root of the Sum of the Squares of the horizontal Diſtance, and Object's perpendicular Height, add the Object's perpendicular Altitude; half of this Sum is the Impetus.

Example.

Suppoſe the perpendicular Height of a Caſtle was 690 Feet, and the horizontal Diſtance 5820 Feet, required the leaſt Impetus to reach the Caſtle?

The Square of 5820=33872400
The Square of 690= 476100

The Square Root of this Sum 34348500 is 5860 and $\frac{5860+690}{2}=\frac{6550}{2}=3275$ the Impetus required.

PROBLEM XX.

The Angle of Elevation and the greateſt Random of a Piece being given, to find what proper Diſtance it ought to be planted to hit an Object, whoſe Height above the Piece's Level is alſo known.

RULE.

RULE.

As Radius to the Sine of double the given Elevation, ſo is half the greateſt Random to half the horizontal Random, at the ſame Elevation.

And as Radius is to the Cotangent of the Elevation, ſo is twice the perpendicular Height of the Object to a *fourth Number.*

From half the horizontal Random take that fourth Number, and note the Remainder, then a mean Proportional between the Remainder and half the horizontal Random, added to the ſame half Random gives the Diſtance required.

Example.

Admit the greateſt Random of a twenty four Pounder was 11640 Feet, what Diſtance may the ſame be planted to hit a Battery 312 Feet above the Level of the Piece at the Elevation of 42° ?

As Radius — — — — — 10.00000

Is to the Sine of double Elevation 84° 9.99761
So is half the greateſt Random 5820 3.76492

To half horizontal Random = 5788 3.76253

As Radius — — — — — 10.00000

Is to Cotangent 42° — — — 10.04556
So is twice the Object's Height = 624 2.79518

To a fourth Number = 693 — — 2.84074

From half the horizontal Random = 5788
Take the fourth Number = 693

Remainder 5095

To find a mean Proportional between the Remainder 5095, and half the horizontal Random 5788, proceed thus,

To

To Log. 5095 — — —	3.70714
Add Log. 5788 — — —	3.76252
	2) 7.46966

The mean Proportional — — 3.73483 is 5430, being the Number anſwering to the Log. 3.73483.

To half the horizontal Random	=5788
Add the mean Proportional	=5430

The Sum is the Diſtance required 11218 Feet, or 2244 Paces nearly.

PROBLEM XXI.

Given a Piece's Elevation and the Random it makes at that Elevation, on a Plane of a given Aſcent, to find the Direction to hit an Object at a given Diſtance upon the Horizon.

RULE.

From the natural Tangent of the given Elevation, take the natural Tangent of the Plane's Aſcent, and note what Angle correſponds to this Difference in the Tables of the natural Tangents, the Angle laſt found added to the Plane's Aſcent, and the Sum taken from 180° leaves a *third Angle.*

Then as the Sine of the Angle correſponding to the Difference, is to the given Random, ſo is the Sine of the *third Angle* to the horizontal Random at the ſaid given Elevation.

And as the horizontal Random now found, is to the given horizontal Random, ſo is the Sine of double the given Elevation, to the Sine of double the Angle of Direction ſought, half whereof is the Direction required.

Example.

Suppoſe at an Elevation of 35° 40′ on a Plane whoſe Aſcent is 10° 30′ the Random was 5280 Feet, it is required to find what Direction will anſwer my Purpoſe, ſo as to ſtrike an Object whoſe horizontal Diſtance is 5090 Feet?

From natural Tangent of 35° 40′=.71769
Take natural Tangent of 10° 30′=.18533

The Angle correſponding is 28° 2′=.53236

To	28° : 02′	From	180° : 00′
Add	10 : 30	Take	38 : 32
Sum	38 : 32	Remains	141 : 28 third Angle.

As Sine 28° 2′ — — — — 9.67208

Is to 5820 — — — — — 3.76492
So is Sine 141° 28′ — — — 9.79446

To horizontal Random=7715 — 3.88730

As 7715 — — — — — 3.88730

Is to 5820 — — — — — 3.76492
So is double Elevation 71° 20′ — 9.97653

To Sine 45° 37′ double the Angle 9.85415

Or 134° 23′
Half whereof is 22° 48½′ } the Directions required.
or 67° 11½′ }

PROBLEM XXII.

Given the horizontal Diſtance, the Impetus of the Piece, and the Object's Angle of Elevation above the Level of the Piece, to find the Elevation of the Piece to hit the Object.

RULE.

As Radius is to twice the Impetus, ſo is the Tangent of the Object's Elevation to a *fourth* Number; add this fourth Number to the given horizontal Diſtance, and note the *Sum*.

Then as twice the Impetus is to the Coſine of the Object's Elevation, ſo is the *Sum* to the Coſine of an Angle, which added to the Angle between the Zenith and Object, gives the Double of the Complements of the Elevation, to hit the Object.

Example.

There is a Caſtle whoſe horizontal Diſtance is 6000 Feet, which was obſerved to ſtand 10° 20′ above the Level of the Piece, whoſe Impetus was known to be 4500 Feet, required the Cannon's Elevation to hit the Caſtle?

As Radius — — — — —	10.0000
To twice the Impetus 9000 — —	3.95424
So is the Tangent of 10° 20′ —	9.26086
To a fourth Number 1641 — —	3.21510

Then 6000+1641=7641 the Sum.

As twice Impetus 9000 — —	3.95424
Is to the Coſine of 10° 20′ — —	9.99289
So is the Sum 7641 — —	3.88315
To Coſine of 33° 22′ — — —	9.92180

The Angle included between the Zenith and Object is 79° 40′, for 90°—10° 20′=79° 40′.

To	79° 40′	From	90° 00′
Add	33° 22′	Take	56° 31′
2)	113° 02′	Remains	33° 29′ the lower Elevation, conſequently the higher Elevation is 66° 50′ to hit it.
Comp. Elev.	56° 31′		

PROBLEM XXIII.

Given the horizontal Random at a known Elevation, to find the Elevation of the ſame Piece, when planted at a given Height above the Horizon, ſo that it may throw a Ball to the greateſt Diſtance poſſible, without altering the Charge of Powder.

RULE.

RULE.

1. Multiply the given horizontal Diſtance in Feet by twice $16\frac{1}{12}$ Feet, divide this Product by the natural Sine of double the given Angle of Elevation, the Square Root of this Quotient is the Velocity.

2. Multiply four times the Height of the Piece by $16\frac{1}{12}$ and to the Product add the Square of the Velocity, then ſay, as the Square Root of this Sum is to the Velocity, ſo is Radius to the Tangent of the Angle of Elevation.

3. As the Sine of double Elevation of the given horizontal Random is to Radius, ſo is the given Random to the greateſt required.

Example.

At an Elevation of 30° with a given Charge of Powder, the Random of a Piece on the Plane of the Horizon was 1500 Yards, it is required to find the Elevation of the ſame Piece when planted at 44 Yards above the Level of the Horizon, ſo that (the Charge of Powder continuing the ſame) the Ball may fall at the greateſt Diſtance poſſible, on a horizontal Plane, which is alſo here required?

Yards. *Feet.*

1ſt, $1500 \times 3 = 4500 \times 32\frac{1}{6} = 144750.$

nat. Sine.

And .86602)144750(167143.94 the Square of the Velocity, whoſe Square Root is 408.83 Feet, the Velocity.

2d, $132 \times 4 = 528 \times 16\frac{1}{12} = 8492$, and 167143.94 added to 8492, the Sum is 175635.94, whoſe Square Root is 419.08.

Veloc. *Rad.*

Then as 419.08 : 408.83 : : 1 : .97554 the natural Tangent of 44° 18′ the Elevation required.

Sine *Rad.*

3d, As 60° : 1 : : 1500 : 1730.05 the greateſt Amplitude.

PROBLEM XXIV.

The greateſt horizontal Random of a Cannon being known, to find the greateſt Random and Direction, when the ſaid Cannon is planted on a Plane above the horizontal Plane, whoſe Height is alſo known.

RULE.

1. From the Square of the Sum of the greateſt horizontal Random, and the Plane's Height above the Plane of the Horizon, take the Square of the Plane's Height above the Plane of the Horizon, the Square Root of the Remainder is the Random required.

2. Then as the Random now found, is to the Random given, ſo is Radius to the Tangent of Elevation.

Example.

A Cannon planted on an Eminence 200 Feet above the horizontal Plane, whoſe greateſt Random on the Horizon is known to be 6000 Feet, it is required to find the greateſt Random from that Eminence, and the Direction to reach it?

Firſt, $6000+200=6200$ the Sum, and 38440000 is the Square of it, and 200 ſquared is 40000, then

From 38440000
Take 40000

Remains 38400000 whoſe Square Root is 6196 the Random required.

2. Then,

As Random found 6196 — — 3.79211

Is to the Random given 6000 — 3.77815
So is Radius — — — — 10.00000

To Tangent 44° 5′ the Direction — 9.98604

PROBLEM XXV.

Given the utmoſt horizontal Random of any Piece of Artillery, to find at what Diſtance, without altering the Elevation or Charge of Powder, it will be able to ſtrike an Object on a Plane of a given Aſcent.

RULE.

Take the Difference between one fourth of the given Random and the Object's Elevation, find a mean Proportional between the given Random and the aforeſaid

Difference, which add to half the given Random, the Sum will be the Diſtance required.

Example.

If the greateſt horizontal Random of a Piece be 1500 Yards, at what Diſtance (ſuppoſing the Charge of Powder and Elevation to continue the ſame) will it be able to ſtrike an Object elevated 44 Yards above the Plane of the Horizon?

4) 1500 the greateſt Random.

375 a fourth Part of it.
44 Object's Elevation.

331 their Difference.
1500 multiply by the greateſt horizontal Random.

1655
331

496500 whoſe Square Root is 704.6 Yards, the mean Proportional.

Then half of the utmoſt Random is 750 Yards, and 750+704.6=1454.6 Yards, the required Diſtance.

PROBLEM XXVI.

Given the Angle of Elevation of any Object above the Level of the Piece, the horizontal Diſtance, and the Piece's Elevation, required the Impetus to hit the Object?

RULE I.

Find the natural Coſine of the Sum of the Angles which the Line of Direction makes with the Object and the Zenith, and likewiſe the natural Coſine of their Difference, and then

As Difference of theſe Coſines
To the Coſine of the Object's Elevation,
So is half the horizontal Diſtance
To the Height of the perpendicular Projection (or Impetus).

RULE II.

To Radius add the Coſine of the Plane's Elevation, the Arith. Com. of the Coſine of the Piece's Elevation, the Arith. Com. of the Sine of the Difference between the Piece's Elevation and the Object's, and the Logarithm of a fourth Part of the horizontal Diſtance together, a tenth Part of the Sum is the Impetus required.

Example.

There is an Object whoſe Height above the Level of the Piece is 10° 20′, its horizontal Diſtance is 7920 Feet, required the Impetus when the Elevation of the Piece is 41° 20′?

The Difference between 41° 20′ and 10° 20′=31° 0′, its Sine is 9.71184.

To Radius — — — — —	10.00000
Add Coſine 10° 20′ — — —	9.99289
The Arith. Com. Coſine of 41° 20′	0.12443
The Ar. Co. Sine (41° 20′—10° 20′)=31°	0.28816
The Log. of $\frac{7920}{4}=1980$ — —	3.29666
To 5037 the Impetus — — —	2|3.70214

PROJECTIONS

Made on DESCENTS.

PROBLEM XXVII.

Given the greateſt Random, on a Plane of a given Deſcent, to find the greateſt Random on a Plane of a given Aſcent.

RULE.

RULE.

As the Sum of the Radius and natural Sine of the given Aſcent, is to the Difference between the Radius and the natural Sine of the given Deſcent, ſo is the given Random to the Random required.

Example.

The greateſt Random of a Piece made on a Plane whoſe Deſcent was 10° is 8000 Feet, what is her greateſt Random on a Plane which is 5° above the Level of the Piece?

As 1.08715 : ,82636 : : 8000 : 6081 the Random required.

Corollary. Given the Angle of Obliquity, you have the Elevation of the utmoſt Random, by ſubtracting half that Angle from 45°.

PROBLEM XXVIII.

Given the greateſt horizontal Random, to find the greateſt Random on a Plane of a given Deſcent.

RULE.

As Radius leſs the natural Sine of the Plane's Deſcent is to Radius, ſo is the given horizontal Random, to the Random required.

Example.

If the greateſt horizontal Random of a Piece be 7960 Feet, what is the greateſt Random on a Plane, which is 10° below the Level of the Piece?

From Radius — —	1
Take natural Sine 10°	0.174
Remains — — —	.826

Then, as .826 : 1 : : 7960 : 9634 the Random required.

Or you may find the Random by the following

RULE.

The Complement of half the Angle included between the Zenith and the deſcending Plane to 90° is the Elevation.

Then as Tangent of the Elevation is to the Tangent of the Plane's Deſcent, ſo is the given greateſt Random to a *fourth* Number, which added to the given horizontal Random, gives the Random required.

Take the above *Example.*

Then 90°+10°=100° the Angle between the Zenith and Plane, half is 50°, its Complement is 40° the Elevation.

As Tangent 40° — — — — 9.92381

Is to the Tangent 10° — — — 9.24631
So is 7960 — — — — — 3.90091

To 1673 — — — — — 3.22341

Then to 1673
Add 7960

Sum 9633 the Random as above, nearly.

PROBLEM XXIX.

Given the greateſt Random on a Plane of a given Deſcent, to find the greateſt Random on the Plane of the Horizon.

RULE.

As Radius is to the Difference between the Radius and natural Sine of the Plane's Deſcent, ſo is the given greateſt Random, to the greateſt Random on the Plane of the Horizon required.

Example.

If the greateſt Random of a Piece on a Plane whoſe Deſcent is 10°, is 9634 Feet, to find the greateſt horizontal Random?

As 1 : .826 : : 936 : 7960 Feet, the greateſt Random on the Plane of the Horizon.

PROBLEM XXX.

Given the Diſtance of any Object on a Plane of a given Deſcent, and the Piece's Impetus, to find the Direction to hit that Object.

RULE.

RULE.

As Radius is to the given Diſtance, ſo is the Sine of the Object's Angle of Depreſſion to a *fourth Number*, to which add twice the Impetus and note that *Sum*, then as the Object's Diſtance is to *that Sum*, ſo is the *fourth Number* to a *fifth* Number; from the Object's given Diſtance take the *fifth* Number, and note half the Difference.

Then as the Impetus is to the half Difference, ſo is Radius to the Sine of an Angle, half the Sum of the Angle laſt found, and the Object's Depreſſion taken from 90° gives the higher Elevation, and half their Difference the lower Elevation.

Example.

What Direction muſt a Cannon have to hit an Object whoſe Diſtance is 7400 Feet on a Plane whoſe Depreſſion is 6°, and Impetus 3980 Feet?

As Radius — — — — —	10.00000
Is to 7400 — — — — —	3.86923
So is the Sine of 6° — — —	9.01923
To 774 a fourth Number — —	2.88846

Then $(3980 \times 2 =)\ 7960 + 774 = 8734$ the *Sum.*

As $7400 : 8734 :: 774 : 913$ a *fifth* Number.

And $\frac{7400 - 913}{2} = 3249$ the half Difference.

As Impetus 3980 — — — —	3.59988
Is to half Difference 3249 — —	3.51175
So is Radius — — — — —	10.00000
To Sine 54° 44′ — — — —	9.91187

Then $90 - \frac{54^\circ\ 44' + 6^\circ}{2} = 59^\circ\ 38'$ the higher Elevation.

And $\frac{54^\circ\ 44' - 6^\circ}{2} = 24^\circ\ 22'$ the lower Elevation.

PRO-

PROBLEM XXXI.

Given a Piece's Random made at a known Elevation on a Plane of a given Deſcent, to find the Piece's Direction to hit an Object at a known horizontal Diſtance.

RULE.

To the natural Tangent of the given Elevation add the natural Tangent of the Plane's Deſcent, and ſee what Angle anſwers to that Sum, from which take the Plane's Deſcent, the Remainder is a *ſecond* Angle, the Sum of the Plane's Deſcent and *ſecond* Angle taken from 180° leaves a third Angle.

As Sine of the third Angle is to the Sine of the ſecond Angle, ſo is the deſcent Random to a fourth Number; as the fourth Number is to the horizontal Diſtance, ſo is the Sine of twice the given Elevation to the Sine of double the Elevation required.

Example.

There is a Plane whoſe Deſcent is 10°, and the Random of a Piece made on that Plane, at an Elevation of 42°, was 7200 Feet; it is required to find the Direction of the ſame Piece, to hit a Battery on the Plane of the Horizon that is 5880 Feet from you?

To natural Tangent of 42° = .90040
Add natural Tangent of 10° = .17632

The Angle anſwering is = 47° 6′ 1.07672

From 47° : 6′ From 180°
take 10 : 0 take 47 : 6′

Second ∠ 37 : 6 132 : 54 third ∠.

As Sine 132° 54′ — — — — 9.86483

Is to 37° 6′ — — — — — 9.78046
So is 7200 — — — — — 3.85733

To 5928 a fourth Number — — 3.77296

As 5928 — — — — — 3.77296

Is to 5880 — — — — — 3.76937
So is Sine (42°×2=) 84° — — 9.99761

To 80° 31′ — — — — — 9.99402

Then $\frac{80^\circ\ 31'}{2}=40^\circ\ 15\frac{1}{2}'$ the Direction required.

PROBLEM XXXII.

Given the greateſt Impetus of a Piece to find the leaſt Impetus, to hit an Object at a known Diſtance, on a Plane of a given Deſcent.

RULE.

As Radius is to the Sine of the Plane's Deſcent, ſo is the Object's Diſtance to a fourth Number; half the Difference between the Object's Diſtance and the fourth Number, is the leaſt Impetus required.

Example.

Suppoſe the greateſt Impetus of a twenty four Pounder be 6000, it is required to find the leaſt Impetus to hit an Object whoſe Diſtance is 4600 Feet on a Plane 14° 52′ below the Level of the Piece?

As Radius — — — — — 10.00000

Is to the Sine of 14° 52′ — — 9.40920
So is 4600 — — — — — — 3.66275

To 1180 a fourth Number — — 3.07195

Then $\frac{4600-1180}{2}=1710$ the leaſt Impetus.

PROBLEM XXXIII.

Given the Impetus of a Piece, placed on a Plane whoſe Height above the Horizon is alſo given, to find the Elevation to hit an Object at a given horizontal Diſtance.

RULE.

RULE.

As the Object's Height above the Horizon, is to the horizontal Diſtance, ſo is Radius to the Tangent of double the Angle of Elevation, half whereof is the Elevation required.

Example.

Admit the Impetus of a twenty four Pounder be 6000 Feet, planted on a Rampart 410 Feet high, above the Plane of the Horizon, it is required to find the Elevation of the ſaid Piece, to hit a Battery whoſe horizontal Diſtance is 5200 Feet?

As 410	2.61278
Is to 5200	3.71600
So is Radius	10 00000
To Tangent of 85° 29′	11.10322

Then $\frac{85^\circ\ 29'}{2} = 42^\circ\ 44\frac{1}{2}'$ the Elevation required.

PROBLEM XXXIV.

Given the greateſt Random in Feet of any Piece of Artillery, to find the Time of the Ball's Flight.

RULE.

Half the Square Root of one fourth of the greateſt Random in Feet, is equal to the Time of Flight in Seconds.

Example.

Suppoſe the greateſt Random of a twenty four Pounder be 15000 Feet, to find the Time of the Ball's Flight, in reaching the Object?

First, a fourth Part of 15000 is $=3750$.
And the Square Root of $3750=61$.
Therefore $\frac{61}{2}=30\frac{1}{2}$ Seconds, the Time required.

PROBLEM XXXV.

Given the Impetus, Piece's Elevation, and the Elevation of the Object above the Level of the Piece, to find the Time of Flight.

RULE.

Take the Object's Elevation from the Piece's Elevation, and note the *Difference*, and take half the Square Root of the given Impetus in Feet; then ſay, as the Coſine of the Object's Elevation above the Level of the Piece, is to half the Square Root of the Impetus, ſo is the Sine of the Difference, to the Time of Flight in Seconds.

Example.

There is a Plane ſtanding 12° 30′ above the Level of my Piece, on which is ſituated a Caſtle, I want to adjuſt a Fuſe for a Bomb to ſtrike the Caſtle, when the Impetus of the Piece is 4900 Feet, and the Elevation is 44° 30′, required the Time of Flight?

From the Piece's Elevation 44° : 30′
Take the Object's Elevation 12 : 30

The Difference is — — 32 : 00

The Square Root of 4900 is 70, and half 70=35.

As Coſine 12° 30′ — — — 9.98958

Is to 35 — — — — — 1.54406
So is the Sine of 32° — — — 9.72421

To 19 Seconds — — — — 1.27869

The foregoing Problems ſeem to be the moſt material that can occur in the Firing of Artillery, and by what is already done, may other Problems be anſwered that ſhould happen in Practice, for hitting Objects, whether on a level Surface, or placed above or below the Level of the Piece.

When any Object is to be bombarded on a horizontal Plane, Engineers generally elevate the Piece to 45°, and then increaſe or diminiſh the Charge of Powder, till the Object aimed at can be hit; and by this Method of proceeding much Powder is ſaved; for at 45° Elevation the Velocity is leſs, and conſequently requires a leſſer Charge of Powder for a horizontal Random, than at any other Elevation; if the Piece be levelled below 45 , and the Bomb does not fall to the Diſtance required, the Piece muſt be raiſed a little, but if the Bomb falls beyond the Object aimed at, it muſt then be lowered a little, and by theſe Trials the proper Elevation may eaſily be acquired. When you have found the proper Elevation, then fix it in that Direction, and you will throw very near the Object, I ſay very near, for there often happen Circumſtances, which alter the Flight of the Bomb, becauſe it is ſome difficulty to charge a Mortar in a hurry, exactly in the ſame manner, ſo as to make it correſpond with ſtrict mathematical Rules.

When any Buildings are to be bombarded, it is better to elevate the Piece above 45 Degrees, than under it, for then the Bomb riſes to a greater Height, and conſequently falls with a greater Force, and will do more damage; and the greater the Charge the better they ſucceed; but more than ſufficient to burſt the Bomb, cannot produce any advantageous Effect.

But when it is required to throw Bombs amongſt a Parcel of Men, level the Mortar below 45 Degrees, for then the Bomb will have leſs force, and will not go ſo far into the Ground, and the Splinters occaſioned by its Exploſion will do more Execution; but increaſing or leſſening the Charge of Powder is better, and then you will have no need to alter the Elevation.

The greateſt Diſtance to which a Bomb of five or ſix hundred Weight can be thrown with the ſtrongeſt Charge, is between two and three Miles.

THE Problems propoſed in the foregoing Pages being ſolved in a practical manner, without ſhewing the Inveſtigation of their ſeveral Rules from the Theory, I thought to have added no more, but to pleaſe ſome Part of my Readers, who deſire to ſee into the Theory of Projectiles, I ſhall here inſert from the *Harmonia Menſurarum* of the celebrated Mr *Cotes*, an *Engliſh* Tranſlation of what he has delivered at Page 87, concerning the Motion of Projectiles in a non-reſiſting Medium, *viz.*

THEOREM.

The Curve deſcribed by the Motion of a Body projected obliquely is a Parabola, and the Velocity of a Projectile in any point of the Parabola is ſuch, as a Body acquires in falling down the fourth Part of the Parameter belonging to that Point taken from the Vertex.

Suppoſe the Projectile be thrown from A to E, (*Fig.* 3.) that is, let A E be the Line of Direction, and A C D be the Curve deſcribed: Let A E be the Space which the Projectile would deſcribe in any given time, by means of the Force impreſſed, ſuppoſing it had no Motion downward from the Force of Gravity. And let A B be the Space through which it would deſcend from A towards the Center of the Earth in the ſame Time. By compleating the Parallelogram E A B C, it will eaſily appear that the Projectile by the Compoſition of Motion at the End of the given Time will be found in the Point C. But A E is as the Time in which it is deſcribed, conſequently $\overline{AE}^2$ or $\overline{BC}^2$ is as the Square of the Time (by Th. 2. deſcent of heavy Bodies) therefore $\overline{AE}^2$ or $\overline{BC}^2$ is as A B, wherefore the Points C of the Curve deſcribed through which the Projectile moves are in the Parabola, whoſe Diameter is AB, Vertex A, and the Parameter belonging to the Vertex A is $\frac{\overline{BC}^2}{AB}$ or $\frac{\overline{AE}^2}{EC}$.

The Velocity of a Projectile in any Point A is ſuch as would carry it from A to E in the ſame Time that a Body would deſcend from E to C, and the Velocity acquired by this deſcent from E to C is to that Velocity whereby the Length A E would be deſcribed in the ſame Time as EC to $\frac{1}{2}$A E (by Theo. 1. deſcent of heavy Bodies) wherefore the Velocity acquired in falling from E to C, is to the Velocity acquired in falling down the fourth Part of the Parameter, namely $\frac{\frac{1}{4}\overline{AE}^2}{EC}$, in the ſubduplicate Ratio of the Length EC to the Length $\frac{\frac{1}{4}\overline{AE}^2}{EC}$ (by Theo. 2. deſc.

Bodies) that is, as the Square Root of E C to the Square Root of $\overline{\frac{\frac{1}{4}AE^2}{EC}}$, which Square Roots are to one another as E C to $\frac{1}{4}$A E, wherefore the Velocity of a Projectile in the Point A, is ſuch as a Body would acquire in falling through a fourth Part of the Parameter belonging to the Vertex A.

Q. E. D.

Corollary 1. If a Projectile be thrown according to the Direction A E, and the Parameter of the Parabola which a Projectile deſcribes belonging to the Vertex A be equal to $\frac{AE^2}{EC}$, this Parabola will paſs through the Point C.

Corollary 2. In the ſame or different Parabolas, the Parameters belonging to as many Vertices, are to one another in the duplicate Ratio of the Velocities, wherewith Projectiles are carried in thoſe Vertices (by The. 2. deſc. Bodies).

Corollary 3. Wherefore in any Parabolas whereſoever, if the *Impetus*'s are equal, or the Motion of the ſame Projectile, or equal Velocities of different Projectiles, the Parameters will always be equal, and *vice verſâ*, howſoever different the Direction of the Motion ſhall be in thoſe Places.

PROBLEM.

Given the Velocity wherewith a Projectile is thrown from any given Place, and alſo the Poſition of a Mark, to find the Directions to hit that Mark.

The Velocity being given there will be given the Parameter of the Parabola which the Projectile deſcribes For it is manifeſt by the above Theorem, if a Body deſcends ſo far whilſt its given Velocity in falling be acquired, the Height by this deſcent will be equal to a fourth Part of the Parameter.

Let C be the Mark ſituated in the given right Line AC, (*Fig.* 4.) at A, the Place from whence the Projectile is to be thrown, erect A P perpendicular to the Horizon, and equal to the Parameter found at the Point A, (for the Parameter is given, ſince the Velocity wherewith the Projectile is caſt from the Point A is given; for it is equal to four times the Height from which a Body muſt fall in order to acquire that Height.) Let A P be biſected by the Line K H, cutting it perpendicularly in G, at A erect A K perpendicular to A C, and continue it till it meets in K, from K as a Center, and with the Diſtance K A, deſcribe the Circle A H P. Laſtly, let the right Line B C E I be drawn perpendicular to the horizontal Line A B, ſo as to paſs through the Mark C, and if poſſible to cut the Circle in E and I; A E and A I will be the Directions ſought, in either of which the Projectile being thrown with the given Velocity will hit the Mark.

Q. E. D.

For if P E, P I be joined by reaſon of the ſimilar Triangles P A E, A E C, P A will be equal to $\frac{AE^2}{EC}$, for $PA : AE :: AE : EC$, and $PA \times EC = AE \times AE$, therefore $PA = \frac{\overline{AE}^2}{EC}$. Likewiſe by reaſon of the ſimilar Triangles P A I, A I C, P A will be equal to $\frac{\overline{AI}^2}{IC}$, for $PA : AI :: AI : IC$, whereſore, ſince P A is equal to the Parameter at the Point A of the Parabola which the Projectile deſcribes, it is evident by Corollary 1. of the above Theorem, that this Parabola will paſs through the Mark C.

Corollary 1. It is manifeſt that the right Line A H biſects the Angle C A B, which meaſures the viſible Diſtance between the Mark and the Zenith.

Corollary 2. When the Points E, I, coincide in H, or the two Directions A E, A I coincide into one Line A H, then A C the Diſtance of the Mark becomes A K, a *maximum*, that is the greateſt Diſtance or Random that can be paſſed through with that given Velocity.

Corollary 3. Any two Directions A E, A I either of which may be made to the ſame Mark, are equally diſtant from the Direction A H, that is, the Angles E A H, I A H are equal.

Corollary 4. The horizontal Diſtance of the Mark A C or A B, is as the Sine of double the Angle of Elevation E A C or I A C. For if the right Line C E I meet the right Line G H in F; A C or G F will be equal to the Sine of the Arc A E or A I, which meaſures double the Angle E A C or I A C (*Fig.* 5.)

Corollary 5. But Aϰ the greateſt Diſtance of the Mark ſituated in the horizontal Line Aβ is equal to AG half of the Parameter, and conſequently is given, and this Mark ϰ may be hit, when HAϰ the Angle of Elevation, is half a right one, or 45 Degrees.

Corollary 6. If a Projectile be thrown according to the Direction of AE, the Altitude of the higheſt Point of the Parabola which the Projectile deſcribes above the Horizon, will be equal to $\frac{1}{4}$EC. For if AC be biſected in T, and TR be drawn parallel to CE, cutting AE in R; AC will be an ordinate to TR, the Axis of the Parabola, which its principal Vertex will divide into two equal Parts in V; wherefore the Altitude TV is equal to $\frac{1}{2}$TR or $\frac{1}{4}$C E, but when CE is the verſed Sine of the Arch AE, or of double the Angle CAE, it is manifeſt that TV, the Altitude of the Projectile is the verſed Sine of double the Angle of Elevation.

Corollary 7. But the greateſt Altitude of a Projectile thrown according to the Direction AP, perpendicular to the Horizon, will be equal to $\frac{1}{4}$ of the Parameter AP, and conſequently is given; for in this Caſe AE, EC coincide with the perpendicular Direction AP, the Altitude TV, or $\frac{1}{4}$EC, will now become $\frac{1}{4}$AP.

Corollary 8. The Time wherein a Projectile, thrown according to the Direction AE, will be carried from A to C, is as the Chord AE or as $\frac{1}{2}$AE, that is, as the Sine of the Angle of Elevation EAC.

Corollary 9. But the greateſt Time when a Projectile is thrown according to the Direction AP, which is perpendicular to the Horizon, is the ſame as a Body will deſcend from P to A, through the whole Parameter, and conſequently is given. For in this Caſe the Projectile aſcends in a right Line to the Altitude $\frac{1}{4}$AP, that is, will riſe to a fourth Part of the Parameter, and then deſcends from the ſame Altitude; and the Times of Aſcent and Deſcent taken together, make twice the Time of the Deſcent alone.

Since the Writing of the above, Mr *Emerſon*'s *Principles of Mechanics* came into my Hands, where I find at Page 33 to Page 37, he has elegantly treated the Subject of Projectiles, and ſeems to have rendered that Subject quite compleat, by adding a new Propoſition to the other, from which he deduces two general Scholia which anſwer all the uſual Queſtions in the Doctrine of Projectiles: for all the foregoing Problems I compared with his Scholia, and I found the Anſwers exactly agreed, though quite different in their Solution.

The firſt is deduced from his general Propoſition *of horizontal Diſtances of Projections, made with any Velocities, and at any Elevations*, where for brevity's ſake he puts

v = *Velocity of the Projectile, meaſured by the Space it paſſes through in Time* 1.
f = *deſcent of a Body by Gravity in the ſame Time.*
x = AE, *the horizontal Diſtance, or Amplitude.*
s = *Sine* } *of the Elevation* VAE.
c = *Coſine* } *of the Elevation* VAE.
A = *Sine* } *of twice the Elevation.*
B = *verſ. Sine* } *of twice the Elevation.*
h = *Height of the perpendicular Projection with the Velocity* v, (*Fig.* 6.)

SCHOLIUM.

Then $h=\frac{vv}{4f}$ whence

Horizontal Diſtance $=\frac{vvsc}{f}=\frac{vvA}{2f}=4sch=2Ah.$

Altitude of the Projection $=\frac{vvss}{4f}=\frac{vvB}{8f}=ssh=\frac{1}{4}Bh.$

Time of Flight $=\frac{vs}{f}=2s\sqrt{\frac{h}{f}}.$

From the *Data* of any Queſtion in Projectiles, may the unknown Quantities be found by thoſe that are known by an eaſy Reduction As for Inſtance this:

PROBLEM XXXVI.

Given the horizontal Diſtance at a known Elevation, to find the Piece's Impetus, or perpendicular Projection, Altitude of Projection, Time of Flight, and Velocity of the Ball.

Example.

Let a Ball be projected at an Angle of 32 Degrees Elevation, whoſe horizontal Diſtance was found to be 2000 Feet, what is the Piece's Impetus, Altitude of Projection, Time of Flight, and Velocity?

Here $s = .52991$ the nat. Sine } of 32° the Elevation.
$c = .84804$ the Coſine }
$f = 16\frac{1}{12}$ Feet, the Space a Body falls in 1 Second.
$x = 2000$ Feet, the horizontal Diſtance.

Then	1	$4sch = x$ by the Scholium.
$1 \div 4sc$	2	$h = \frac{x}{4sc} = 1112.6$ the perp. Shot, or Impetus.
But	3	$ssh = 312.4$ the Altitude of Projection.
And	4	$2s\sqrt{\frac{h}{f}} = 8.79$ Seconds, the Time of Flight.
Alſo	5	$\frac{vv}{4f} = h$.
$5 \times 4f$	6	$vv = 4fh$.
6ɯ2	7	$v = \sqrt{4fh} = 267.53$ Feet in a Second, the Velocity.

The other Scholium is deduced from his general Propoſition of *the Diſtances of Projections made on inclined Planes.*

Where AB denotes the inclined Plane, (*Fig.* 7.) AC the Direction of the Projectile, AZ the Zenith Line, AEB the Path of the Ball, and let

v = *Velocity of the Projectile in* A, *meaſured by the Space it deſcribes in the Time* 1.
f = *Space deſcribed by a deſcending Body in the ſame Time.*
s = *Sine of* CAB.
c = *Sine of* CAZ.
z = *Sine of* ZAB.
x = AE *the oblique Diſtance or Random.*
h = *Height of the perpendicular Projection,* or Impetus as before.

SCHOLIUM.

Impetus or *perpendicular Projection* $h = \frac{vv}{4f}$. Whence

Length of the Projection $= \frac{scvv}{fzz} = \frac{4sch}{zz}$.

Height of the Projection $= \frac{ssvv}{4fzz} = \frac{ssh}{zz}$.

Time of Flight $= \frac{sv}{fz} = \frac{2s}{z}\sqrt{\frac{h}{f}}$.

For an Illuſtration of the above, take the following

PROBLEM XXXVII.

Given the utmoſt Random on an aſcending Plane, to find the Impetus, Height of the Projection, Time of Flight, and Velocity.

Example.

Suppoſe the Length of the Projection on an aſcending Plane is 6840 Feet, being the utmoſt Random of a Cannon, to find the Height of the perpendicular Projection, the Height of the Projection, Time of Flight, and Velocity.

Here ſince ZAC=CAB, therefore

$$x=\frac{4ssh}{zz},$$

$$h=\tfrac{1}{4}x\times\frac{zz}{ss}=x\times\frac{s^2, \mathrm{CAD}}{\mathrm{Rad.}^2}$$

$$v=\sqrt{4fh}$$

$\frac{ssh}{zz}=\frac{1}{4}x$, *Height of the Projection.*

$\frac{2s}{z}\sqrt{\frac{h}{f}}=\sqrt{\frac{x}{f}}$ *the Time of Flight.*

As I have been very particular in illuſtrating the foregoing Problem in Numbers, I ſhall not inſiſt any more on this, it being ſo eaſy to be underſtood.

Notwithſtanding what has already been advanced on this Subject, the knowing the true Diſtance that any Piece will throw a Bullet at all times, is a Matter ſubject to many Uncertainties, ſince there is ſuch a Variety in the trueneſs of the Bore, in the Levelling and Directing the Piece, the Nature of the Powder taking Fire, and the Reſiſtance of the Air, which laſt being a reſiſting Medium renders the Curve of the Shot nearer an Hyperbola than Parabola; and if any Perſon conſiders the Curve as ſuch, as is laid down in the *Principia* of the illuſtrious Sir *Iſaac Newton*, ſo as to apply it to Practice, would in my Opinion take upon him a very hard Taſk; for it is one of the moſt difficult Problems in all the Doctrine of Fluxions, to ſhew how far a Projectile deviates from a parabolic Track, by the Reſiſtance of the Air; for the Principles and Calculations made uſe of on this Occaſion, will in no wiſe be fitting to the Experience and Underſtanding of Learners; and what makes it worſe ſtill, it can be of no manner of uſe in the Art of Gunnery, becauſe it can be reduced to no Rules; therefore I have endeavoured to render the Subject as plain as I could to Beginners; I could indeed have added ſome Problems, by ſuppoſing the Bullet to move in a right Line to a certain Diſtance from the Mouth of the Cannon, and then afterwards to deſcribe the Curve of the Parabola; but as theſe are not diſpatched in a few Words, I have purpoſely omitted them, in order to make room for the following Tables, computed from Sir *Iſaac Newton*'s Principles, and applied to the Motion of Projectiles by *D. Bernoulli*, in a *Latin* Memoir printed in Tom. II. Page 338, *&c.* of the *Comm. Acad. Petrop.* I thought (ſays he) no ſmall Profit would occur to theſe Speculations, if I reduced to a Calculation ſome of the Experiments made by the excellent D. *Gunther* before ſome Academicians, whence it will appear what ſtupendous Effects the Air has upon Bodies of near eight thouſand Times its ſpecific Gravity. And from hence the Forces of Gunpowder may be exactly compared among themſelves; and many other uſeful Things deduced. There were abundance of Experiments made of divers kinds, among which I ſhall pick out thoſe that can be ſerviceable in this Affair; ſuch as thoſe made with Guns exactly plac in the Perpendicular, and obſerving the Quantity Powder, and the Time of the Globe's aſcending and deſcending. The Diameter of the Ball was 23¾ hundred Parts of an *Engliſh* Foot, and made of Iron; its ſpecific Gravity to that of Air, as 7650 to 1, ſuppoſing the Air all of the ſame Denſity; the Length of the Bore was 7.7 Feet, whence the following Experiments were made.

Quantity of Gunpowder in Ounces *Holland*.	Whole Time of Aſcent & Deſcent.	Height of the Globe in reſiſting Air.	Time of Aſcent in reſiſting Air.	Time of Deſcent in reſiſting Air.	Height to which the Globe would aſcend in vacuo with the ſame Force.	Time of riſing and falling in vacuo, when projected with the ſame Force.
$\frac{1}{2}$	11	486	5.42	5.58	541	11.6
2	34	4550	14.37	19.63	13694	58
4	45	7819	16.84	28.16	58750	121

* * *

The following Experiments were made with a Gun juſt ſix Feet the Length of the Bore, and with the ſame Globe.

Ounces of Pow-der.	Number of Seconds that the Globe ſtaid in the Air by Obſervation.	Height to which the Globe aſcended in Feet by Calculation.	Time of Aſcent in Seconds by Calculation.	Time of Deſcent in Seconds by Calculation.	Height in Feet that the Globe would aſcend with the ſame Force in vacuo by Calculation	Time of riſing and falling in vacuo, with the ſame Force in Seconds.
½	8	257	3.95	4.05	274	8 2
2	20.5	1665	9.74	10.76	2404	24 5
4	28	3187	12.5	15.5	6604	40.5
6	32 5	4304	13.9	18.6	11810	54.3
8	38	5643	15.54	22.46	22394	74

Both theſe Tables were exactly calculated; we ſhall juſt deduce ſome of the principal Conſectaries therefrom. The greateſt Time of Aſcent was 16″.84, and in the next, the greateſt Time of Aſcent was 15″.54, the Difference is only 1.3″, and yet the Difference of Heights to which the Globe would aſcend in the reſiſting Air is not to be neglected, as amounting to 2176 Feet; from this alone it is plain to any body, that there is a certain Time which the Globe, though aſcending ever ſo high, can never exceed; and the greateſt Time for the Aſcent in our Globe is = 19″.27, whence is confirmed the following

THEOREM.

The Time of Aſcent and Deſcent taken together, is greater in vacuo *than in* pleno, *the initial Velocities being the ſame in both; likewiſe it may be infinite in both Caſes, but yet infinitely greater in* vacuo *than in* pleno.

That the Time of Aſcent and Deſcent of a Globe projected upwards with an infinite Force in a uniform reſiſting Medium, is infinitely leſs than when projected upwards with the ſame Force in *vacuo*, though the Forces of Gravity be the ſame.

Suppoſing the Forces of Exploſion, or the firſt Velocities to be equal and infinite, then will alſo the Height to which the Globe will ariſe in *vacuo*, be infinitely greater than the Height it can ariſe to in a reſiſting Medium; and yet both of them is here infinite.

Theſe Aſſertions have been partly demonſtrated already in the preceeding Paragraph; but it is worth notice. that the Air which we ſuppoſed 7650 Times lighter than Iron, ſhould exert ſo great a Reſiſtance upon an Iron Ball, that only ſtaying 45″ in the Air, it ſhould deſtroy $\frac{7}{8}$ of the Height of its Aſcent by that Force.

From the foregoing Tables we may alſo gather ſome Things concerning the Force of Gunpowder. Here the Forces impreſſed upon Balls are as the Heights to which the Globes would aſcend in *vacuo*. In the firſt Table we find theſe Heights to be as 13694 to 541, or nearly as 26 to 1; when the Quantities of Powder were as 4 to 1. Whence it appears, that the Strength of Powder in driving a Ball, increaſes far faſter than in proportion to the Weight of the Powder; and I can ſee no other reaſon for it than this, that a great Part of the inflamed Powder flies through the Touch hole, and between the Ball and the Barrel; and that this uſeleſs Part of the Powder is in a far leſs Ratio, when a great deal is fired, than when only a little. But this may be collected from Experiments; conſidering the Powder as Air exceedingly condenſed, that is, that force of Air very much condenſed increaſes in a much greater Ratio than the Denſity; for by Calculation I found, that Air 500 Times denſer than in its natural State, was at leaſt 6000 Times ſtronger than the Elaſticity of the common Air.

I ſhall here annex ſome other Experiments made with a Mortar.

The Diameter of the Mortar = 1.05 Feet *Engliſh*.
Diameter of the Ball = 1.01 Feet.
Weight of the Ball = 200 Pounds *Holland*.

Quantity of Powder in Pounds *Holland*	$\frac{1}{2}$	1	2	3	4	5	6
Time in Seconds the Globe ſtaid in the Air.	3	7.5	12	16.5	20	23	25

Thus far D *Bernoulli*; as he never meddles with Projectiles I thought his Memoir but of ſmall ſervice here, becauſe it is only upon perpendicular Shots, but I thought his Tables of Experiments might be of ſome ſervice.

I ſhall here obſerve this is the only Perſon, Mr *Robins* ſays at Page 51 of his Preface, that has conſidered the Actions of the Air on military Projectiles, founded on *Newton*'s Doctrine, but I can ſee very little in *Bernoulli*'s Memoir worth reading. For in the firſt Part he lays down ſeveral Hypotheſes concerning the Preſſure of Fluids, and by ſome Experiments he ſays, he comes at laſt to this, that the Preſſure of any Column of a Fluid is juſt equal to the Weight of it; at the End he calculates the Reſiſtance of a Curve or Solid in a Fluid, in a Curve he ſays it is = Fl. $\frac{p\dot{x}^3}{b\dot{s}^2}$ and in a Solid, that the Reſiſtance is to the Reſiſtance of the Baſe as the Fluent of $\frac{npx\dot{x}^3}{ba\dot{s}^2}$ to $\frac{npx^2}{2b}$; according to our way of Notation : But the whole Memoir is wrote in the differential Method, in an unintelligible manner. Now all this firſt Part is elegantly calculated at Page 254 of Mr *Emerſon*'s *Fluxions*. But *Bernoulli* abſurdly calls Reſiſtance by the Name of Preſſure.

In the ſecond Part he ſpends a deal of Time in finding the greateſt Velocity a deſcending Globe can acquire in a Fluid, and comes to the ſame Concluſion as Sir *Iſaac Newton*, Cor. 2. Prob. 38. B. II. Then afterwards he propoſes to find the Reſiſtance of Ships in the Water, in the finding of which he takes as many Things for granted as he thinks fit, but would have the Reſiſtance found by Experiment of Men drawing a Ship ; all the Calculations he here gives contain nothing but what is common ; this Part he ends by exploding *Carteſius*'s Vortices, and the Deſcent of Bodies upon this Hypotheſis.

The whole Deſign of the third and fourth Parts, is only to find the Motion of Bodies aſcending and deſcending in a Fluid ; and this has been done by a great many People, eſpecially at Page 288 of *Emerſon*'s *Fluxions*, in a more elegant, ſhorter and plainer Method ; I can ſee no uſe any Part of this can be of in the Art of Gunnery, for no Body ever ſhoots perpendicular with great Guns.

Mr *Robins*'s Book, or *New Principles of Gunnery*, is a very good Thing, and the Experiments ſeem to be made with great Care ; but *Bernoulli* differs from him in ſeveral Things, for he makes the Reſiſtance as the Square of the Velocity, but Mr *Robins* makes it more. *Bernoulli* alſo ſays, that the Force of Powder (condenſed as Air condenſed) increaſes in a far greater Ratio than the Denſity ; but Mr *Robins* ſays, Page 7, that its Force is as its Denſity. Thus have I given a ſhort Account of this Memoir of *Bernoulli*'s.

Having a little ſpare Room, I ſhall inſert here a Problem with its Solution, for the Entertainment of my Readers, to determine the Velocity of a Bullet, though of no material uſe at all in the Art of Gunnery.

⁂

PROBLEM XXXVIII.

Given the Dimenſions of any Piece of Artillery, the Denſity of its Ball, and the Quantity of its Charge, to determine the Velocity which the Ball will acquire from the Exploſion, ſuppoſing the Elaſticity of the Powder at the firſt Inſtant of its firing to be given?

SOLUTION.

It has been proved by Mr *Robins*, that the Quantity of the Preſſure exerted by Gunpowder at the Moment of its Exploſion, has an Elaſticity $999\frac{1}{3}$, or in round Numbers 1000 times greater than common Air; and ſince common Air by its Elaſticity exerts a Preſſure on any given Surface equal to the Weight of the incumbent Atmoſphere with which it is in Equilibrio, the Preſſure exerted by fired Powder before it has dilated itſelf, is 1000 times greater than the Preſſure of the Atmoſphere. Now in order to ſolve the Problem, the two following Principles muſt be aſſumed.

I. That as ſoon as the Bullet is got out of the Piece, the Action of the Powder on the Bullet ceaſes.

II. That all the Powder of the Charge is fired, and converted into an elaſtic Fluid, before the Bullet is ſenſibly moved from its Place. Theſe being premiſed

Let AB (*Fig.* 8.) be the Axis of any Piece of Artillery, A the Breech, B the Muzzle, DC the Diameter of the Bore, DEGC a Part of the Cavity filled with Powder.

Let $AF=a$, $AM=x$, $AB=b$, and $v=$Velocity of the Ball at M, $s=16\frac{1}{12}$ Feet, $c=3.1416$, $d=$Diameter of the Bore, or of the Ball, in Inches, $w=$Weight of the Ball in Pounds; then ſince by Experiments the Preſſure of the Atmoſphere upon a ſquare Inch is 14 Pounds, therefore

$$\frac{14000\,acdd}{4}=\text{Force of the Powder at F, and}$$

$$\frac{1}{a}:\frac{1}{x}::\frac{14000cdd}{4}:\frac{14000\,acdd}{4x}=\text{Force of the Pow-}$$

der at M; now by *Schol.* Page 13 of Mr *Emerſon*'s *Mechanics*, $v\dot{v} \propto F\dot{x}$, and in a falling Body the Velocities generated are as the Times, or as Spaces uniformly deſcribed in theſe Times; therefore $2s$ (Space) : $2s$ (Velocity) : : $\dot{x}$ (Space) : $\dot{x}$ = Velocity generated by a falling Body in the Time of deſcribing $\dot{x}$; therefore in the Caſe of a falling Body, writing $2s$, $\dot{x}$, w, for v, $\dot{v}$, F, and in Caſe of the Ball, writing $\frac{14000\,acdd}{4x}$ for F, in the general Proportion ($v\dot{v} \propto F\dot{x}$) it will be $2s\dot{x} : w\dot{x} :: v\dot{v} : \frac{14000\,acdd}{4x}\dot{x}$, therefore $v\dot{v} = \frac{14000\,acdds\dot{x}}{2wx}$, and the Fluentis $\frac{vv}{2} = \frac{14000\,acdds}{2w} \times \text{Log} : x$. But in F, $x = a$; and corrected it is $vv = \frac{14000\,acdds}{w} \times \overline{\text{Log} : x - \text{Log} : a}$ $= \frac{14000\,acdds}{w} \text{Log} : \frac{x}{a}$. And when $x = b$, $v = \sqrt{\frac{14000\,acdds}{w} \times \text{Log}.\, \frac{b}{a}}$, the Velocity of the Ball at the Mouth of the Cannon.

FINIS.

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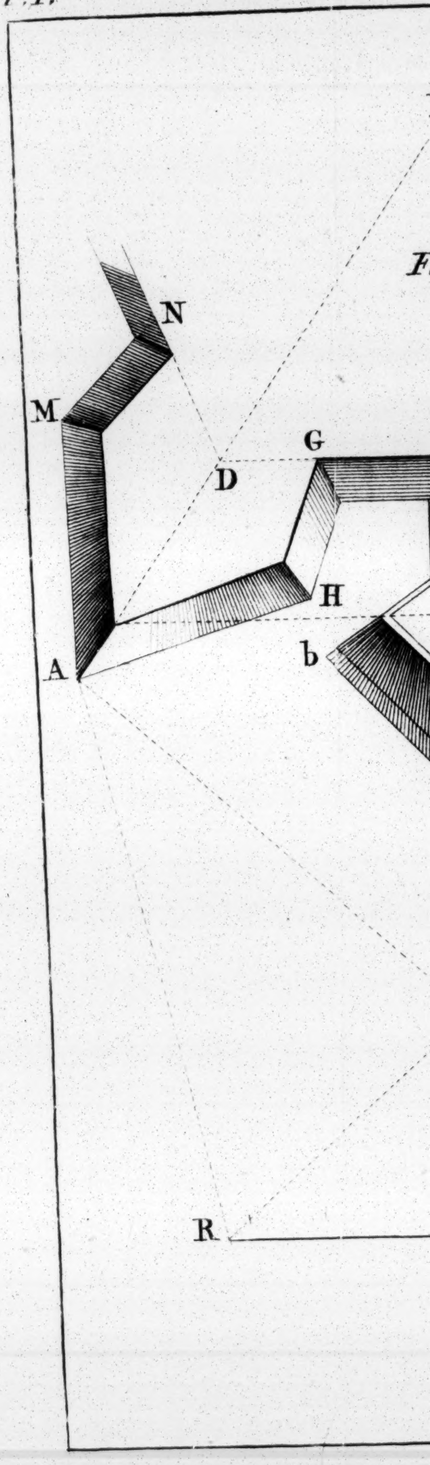

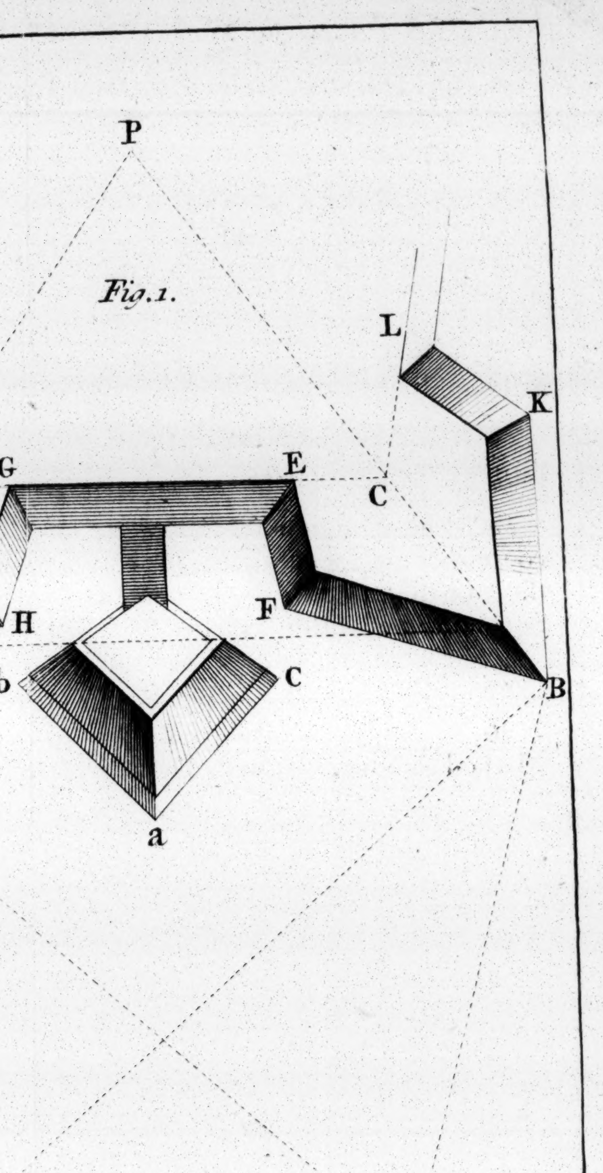
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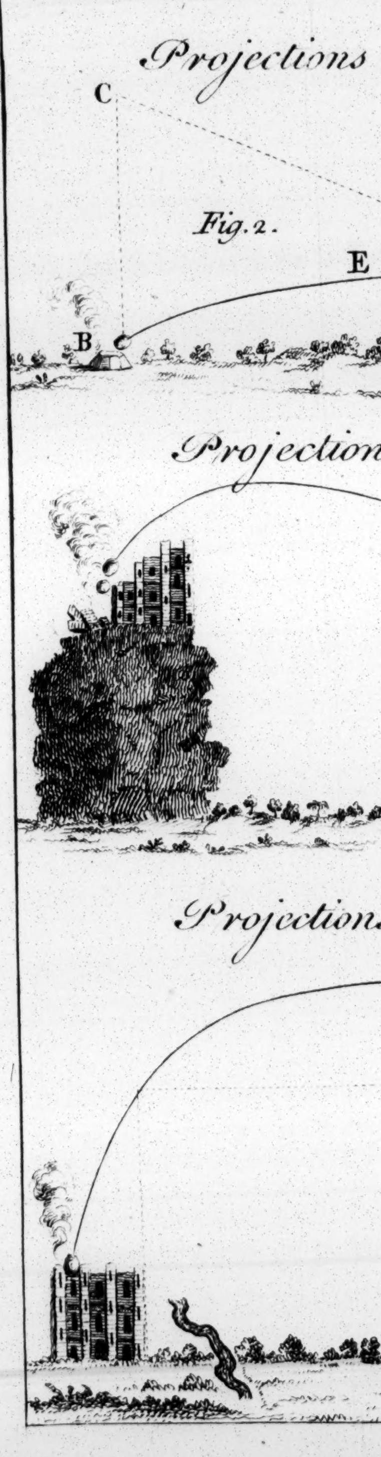

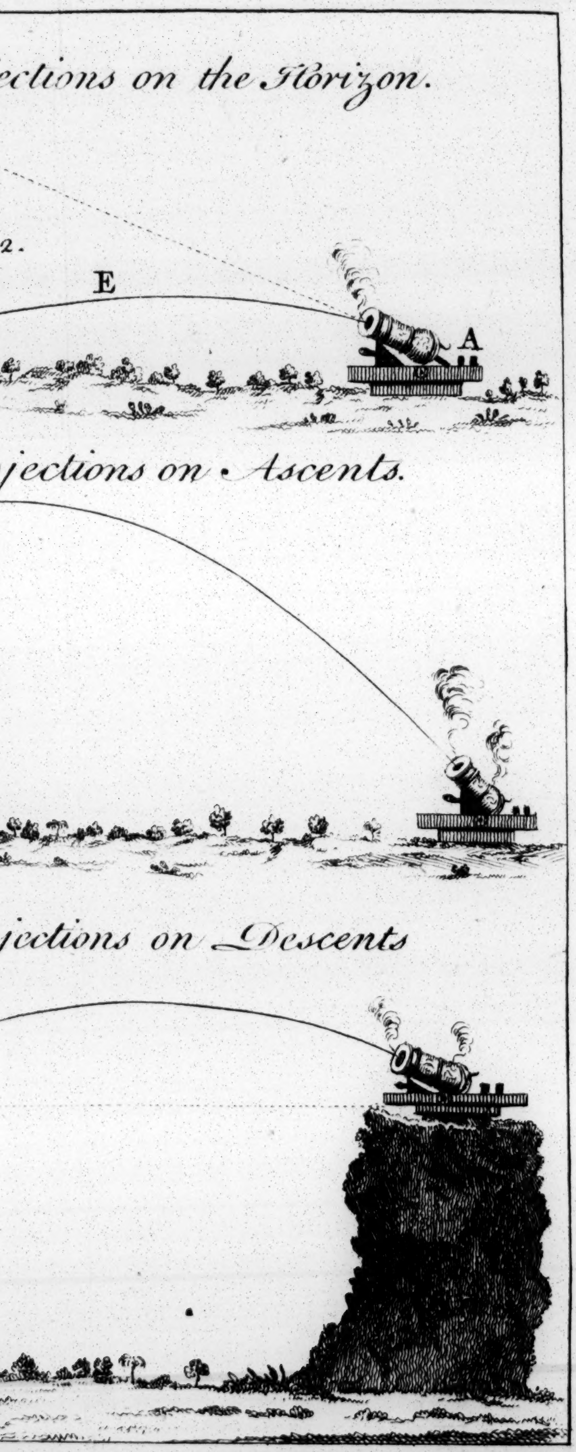
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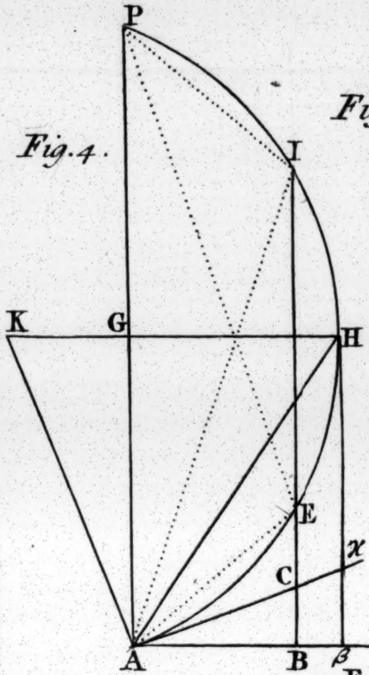

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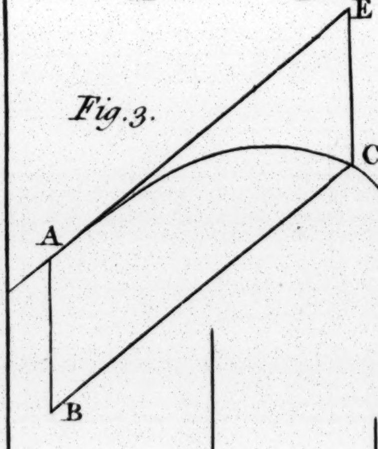

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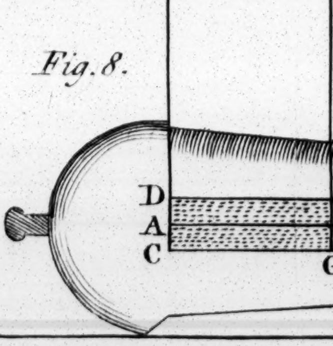

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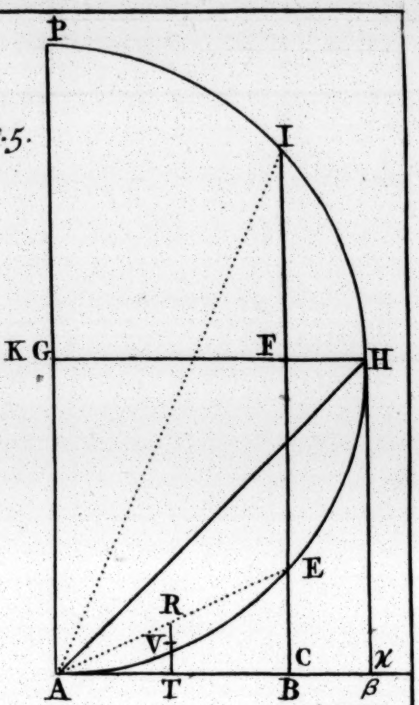

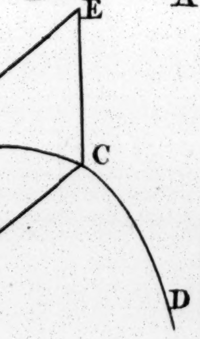

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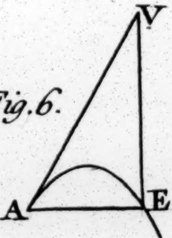

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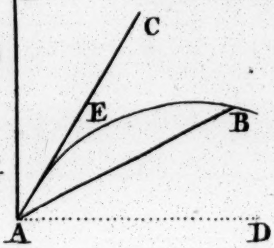

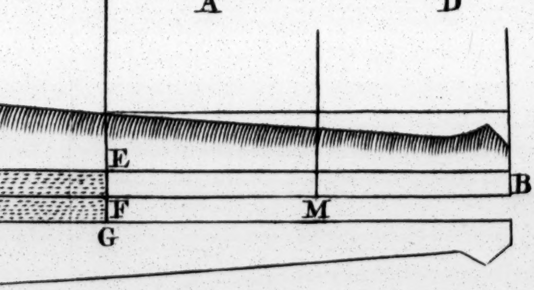

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